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Why Go Spectral?

Harnessing Spectral Pricing Rules in Strategic Portfolio Management

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December 2023

Abstract

"Why Go Spectral? Harnessing Spectral Pricing Rules in Strategic Portfolio Management" delves into the advantages of using spectral (SRM) pricing rules in insurance pricing and planning. Tailored for actuaries engaged in capital modeling, individual risk, reinsurance, and strategic planning, this presentation illustrates how SRM rules not only generalize traditional methods like CoXTVaR but also effectively address their limitations. Instead of prescribing a single solution, SRM methods offer a spectrum of results, each tailored to different risk appetites. Illustrated with a compelling case study, it demonstrates SRM's utility in problems such as diversifying risk pricing and reinsurance evaluation. Readers will acquire the expertise to implement SRM in their work the ability to explain its results to business stakeholders. Incorporate SRM rules in your pricing work to align more closely with your organization's risk appetite and strategic goals!

Introduction to SRM Pricing

Spectral (SRM) Pricing: Overview

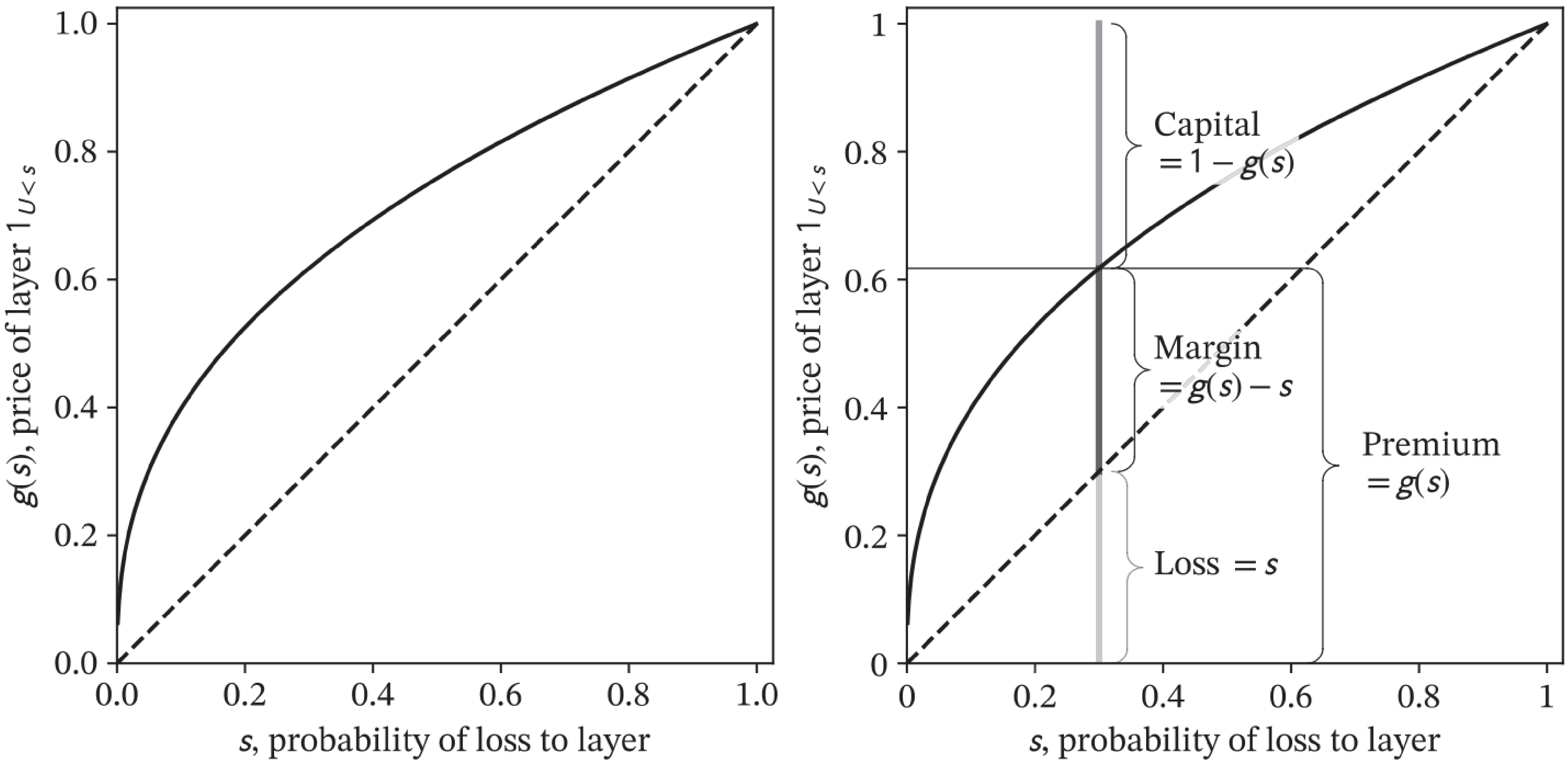
- SRM pricing uses a **distortion function** to add a risk load
- Distortion functions make bad outcomes more likely and good ones less, resulting in a positive loading
- **Distortions express a risk appetite**
- Portfolio SRM premium has a **natural allocation** to individual units
- Many existing methods, including CoXTVaR, are special cases of SRMs

- Different distortions can produce same total portfolio pricing but have materially different natural allocations to units, reflecting distinct risk appetites
- Different allocations, in turn, drive materially different business decisions

Spectral (SRM) Pricing: Distortion Functions

- A **distortion function** g maps a probability to a larger probability, and is used to *fatten the tail*. The distortion function must be
  - Increasing,
  - Concave (decreasing derivative), and
  - Map 0 to 0 and 1 to 1
- $g(s)$ can be understood as the price for a binary risk paying 1 with probability s and zero otherwise
- $S(x) = \Pr(X > x)$, is the survival function of a random variable X
  - Loss cost $\mathsf{E}[X] = \int S(x)dx$
- $g(S(x)) > S(x)$, is the risk-adjusted survival function

Distortion Functions and Insurance Statistics



Graphic: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

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Spectral (SRM) Pricing Rules

- The **spectral (SRM) pricing rule** associated with a distortion g is given by

$$\rho(X) = \int g\left(S(x)\right)dx$$

interpreted as price, technical premium, risk-adjusted loss cost, or risk measure

- Integration by parts trick gives the alternative expression

$$\rho(X) = \int x\, g'\left(S(x)\right)f(x)dx = \mathsf{E}\left[Xg'\left(S(X)\right)\right]$$

which makes the spectral risk adjustment $g'\left(S(X)\right)$ explicit

Spectral Pricing Rules Have Nice Properties

a) **Monotone**: Uniformly higher risk implies higher price

b) **Sub-additive**: diversification decreases price

c) **Comonotonic additive**: no credit when no diversification; if out-comes imply same event order, then prices add

d) **Law invariant**: Price depends only on the distribution

All risk measures with these properties are **SRM rules**

SRM Pricing Adds Up Pricing by Layer

High asset layer: margin high % EL but low % capital; very low leverage

Low asset layer: margin low % EL but high % capital; very high leverage

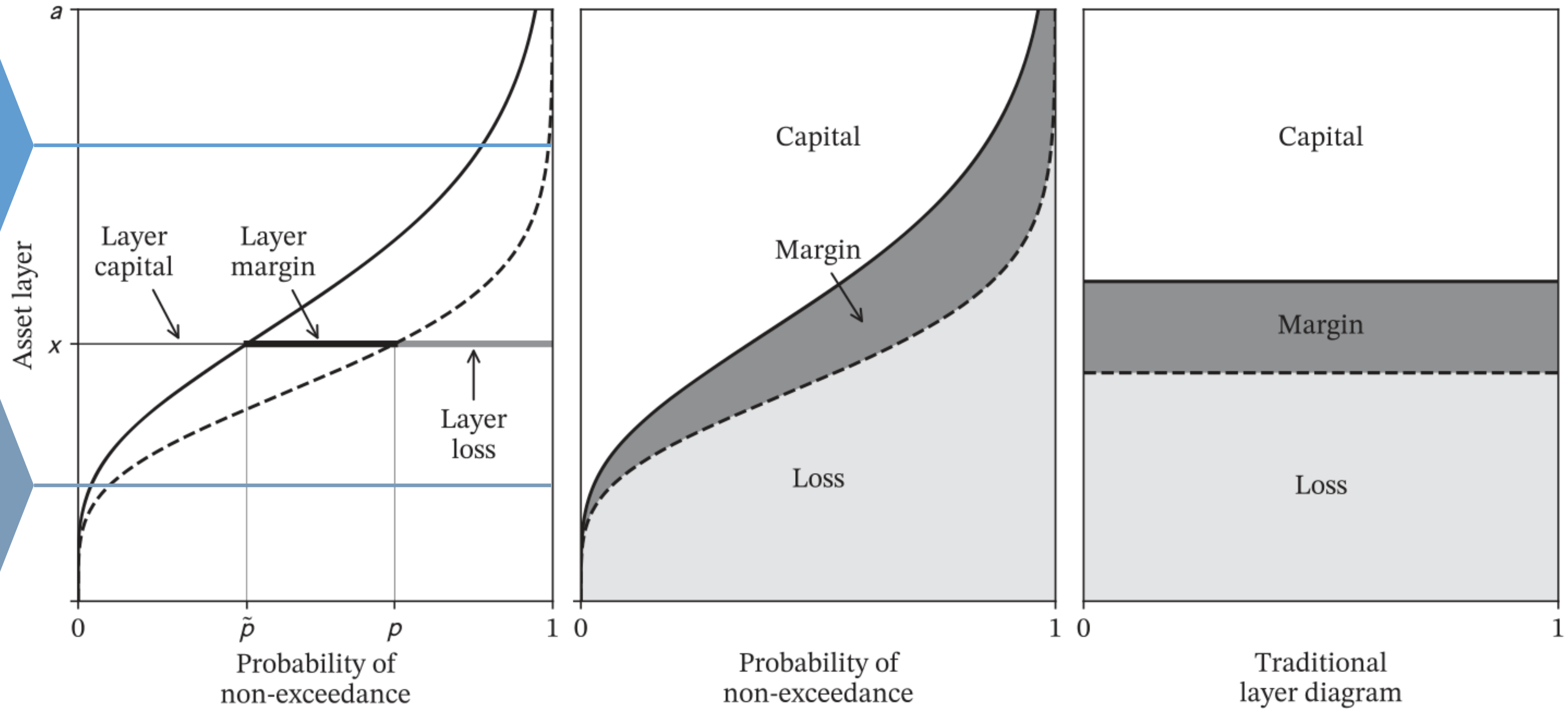

Figure 10.5 Continuum of risk sharing varying by layer of loss (dashed) and premium (solid): by layer (left), in total summed over layers (middle), and traditional (right). The total loss, margin, and capital areas are equal in the middle and right plots. Losses in the left and middle plots are Lee diagrams.

Graphic: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

SRM Pricing has a Natural Allocation to Subunits

- If $X = \sum_i X_i$, define the **natural allocation** to unit i as

$$\mathsf{NA}(X_i) = \mathsf{E}\left[X_i\, g'\big(S(X)\big)\right]$$

- Example: $g(s) = \min\big(1, s/(1-p)\big)$ corresponds to TVaR
  - $\rho(X) = \mathsf{TVaR}_p(X)$
  - $\mathsf{NA}(X_i) = \mathsf{CoTVaR}_p(X_i)$

- The natural allocation pricing has nice properties
  - It is natural because it involves no additional assumptions
  - It adds-up because the sum of natural allocations is the original SRM price
  - It equals marginal pricing when marginal pricing is well defined

Portfolio Pricing

- General portfolio pricing rule

Premium = expected loss + cost of capital

- Cost of capital expressed in dollars, and averages
  - Use of different forms of capital, equity, debt, reinsurance
  - Each with different costs

- Price excluding expenses, investment income, etc.

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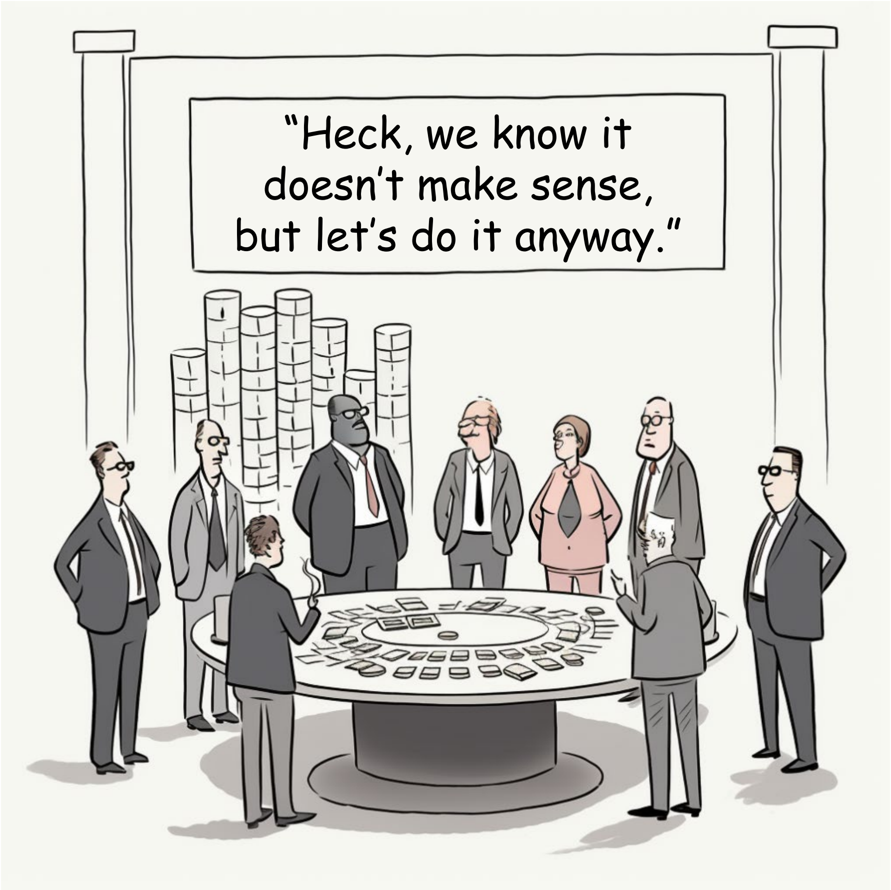
CCoC Portfolio Pricing

- Constant cost of capital (CCoC) is a common default assumption
  - Constant across lines of business
  - Constant across layers of capital (debt, equity, reinsurance, etc.)

- The CCoC of capital r has various names: target return on capital, WACC, opportunity cost of capital

- CCoC Portfolio pricing rule

Premium = expected loss + $r \times$ (amount of capital)



CCoC Critique

Capital use and capital cost vary by layer

- Different costs manifest in WACC calculation!
- Capital allocation methods assume all capital has the same cost
- $r \times$ capital
  = (Avg cost of capital across layers) $\times$ (Avg use of capital across layers)

  $\neq$ Average[(cost of capital by layer) $\times$ (use of capital by layer)]
- Compare $\mathsf{E}[XY] \neq \mathsf{E}[X]\mathsf{E}[Y]$ unless X, Y are uncorrelated

Cost and use are **correlated** because **higher layers are bigger and cheaper,** and cat exposed lines use higher layers disproportionately

CCoC Portfolio Pricing with an XTVaR Capital Standard

- CCoC implementation with XTVaR capital:

$$P(X) = \mathsf{E}[X] + r\,\mathsf{XTVaR}_p(X) = (1-r)\mathsf{E}[X] + r\,\mathsf{TVaR}_p(X)$$

- Rule is a special case of SRM pricing
- Corresponding distortion is

$$g(s) = (1-r)s + r\min\big(1, s/(1-p)\big)$$

  – Weight $1-r$ applied to all events: risk neutral part
  – Weight r applied to p-tail events: extremely risk averse
  – An average of two TVaRs, since $\mathsf{E}[X] = \mathsf{TVaR}_0(X)$
- Easy to check that $\rho(X) = (1-r)\mathsf{E}[X] + r\mathsf{TVaR}_p(X)$ because integrals are linear

XTVaR Natural Allocation

- Corresponding natural allocation is simply CoXTVaR pricing

$$\mathsf{NA}(X_i) = (1 - r)\mathsf{E}[X_i] + r\,\mathsf{CoTVaR}(X_i) = \mathsf{E}[X_i] + r\,\mathsf{CoXTVaR}(X_i)$$

- Shows SRM approach generalizes existing methods

Obvious Question: What about using other distortions?

1. What other distortions are available?

2. How can different distortions be interpreted?

3. Do business implications vary materially by distortion?

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Obvious Question: What about using other distortions?

1. What other distortions are available? **Many others available**

2. How can different distortions be interpreted? **They encode risk appetite**

3. Do business implications vary materially by distortion? **Yes!**

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1. What other distortions are available?

Five **usual suspect** distortions

- **CCoC**: $g(s) = d + vs$ for $s > 0$ and $g(0) = 0$ where $d = 1/(1 + r)$, $v = 1 - d$ are discount rates related to the cost of capital, note $d/v = r$
- **PH** proportional hazard: $g(s) = s^{\alpha}$, $0 \le \alpha \le 1$
- **Wang**: $g(s) = \Phi(\Phi^{-1}(s) + \lambda)$
- **Dual**: $g(s) = 1 - (1 - s)^{\beta}$, $\beta \ge 1$
- **TVaR**: $g(s) = \min\left(1, s/(1 - p)\right)$

- All one-parameter distortions, easy to calibrate to given portfolio pricing
- Many others available, but these five provide a sample good range
- See PIR §11.3 for more details

A word about **TVaR** as a pricing rule

- TVaR usually a risk measure, with p close to 1

- TVaR can be used as a pricing rule, with p commonly between 20% and 60%
  - Rule: simply average worst 40-80% of outcomes

2. How can different distortions be interpreted?

Distortion functions encode risk appetite

- Distortion prices a binary (all or nothing) event with probability s of occurring

Statistic	Risk appetite interpretations
Premium	Price for small s corresponds to tail risk. Mass at zero implies minimum premium.
Loss ratio	Minimum premium implies loss ratio goes to 0 for small s Loss ratio increases to 1 for small (attritional) losses, s=1.
Margin	Symmetric about s=1/2? Skewed left ➔ tail-centric Skewed right ➔ volatility-centric
Return on capital	Bounded or unbounded for equity, on right?
VaR weight	Where is breakeven point, weight=1? Are all VaRs weighted? Is max weighted?
TVaR weight	Masses? Weights 0 or 1 or other values?

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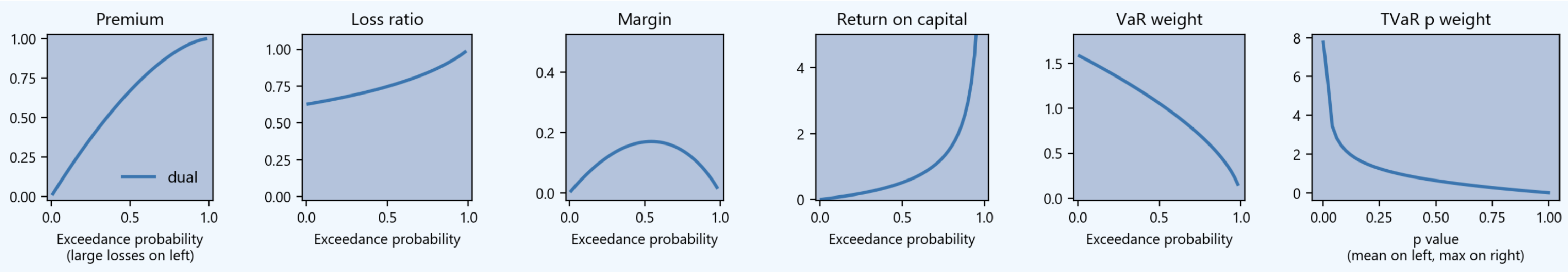
Dual Distortion Insurance Statistics



Premium	Loss Ratio	Margin	Return	VaR weight	TVaR weight
Premium by layer Graph of g Diagonal shows loss cost	Loss / Premium **Look for** minimum LR > 0 vs. LR tending to zero (more tail risk averse)	Premium – Loss Dollar value **Look for** symmetry & location of peak	Return on capital, where capital equals 1 – premium **Look for** cost of equity on the right	All SRMs are weighted averages of VaRs. **Look for** breakeven between over and underweighting.	All SRMs are weighted averages of TVaRs. This graph shows the weights assigned each component. **Look for** wts to mean and max.

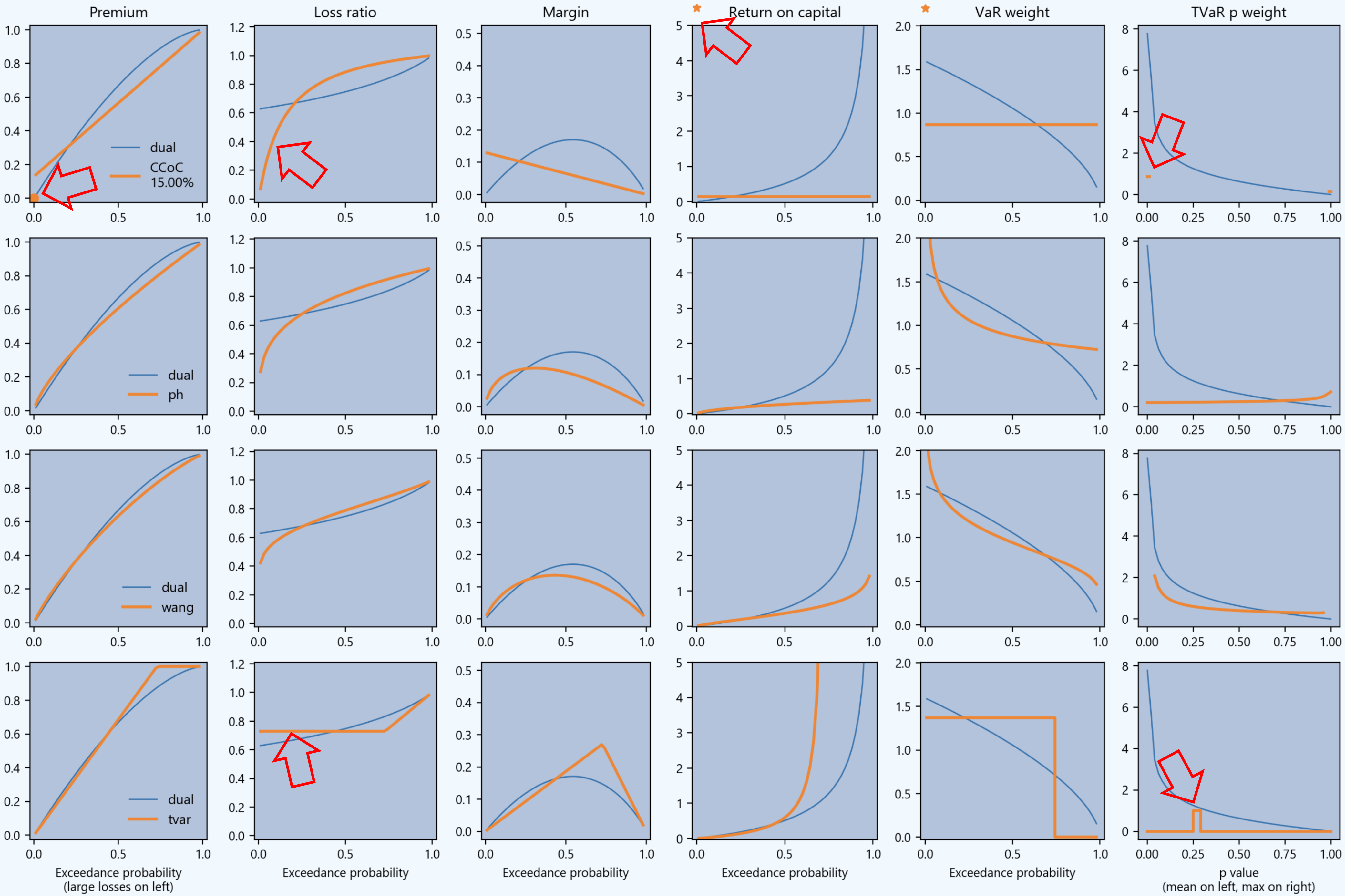
Next slide shows same graphs for Usual Suspects compared to dual shown here; all calibrated to same pricing on a gross portfolio

Calibrated Distortions

- For a given portfolio, calibrate the usual suspect distortions to overall portfolio pricing and compare natural allocation premiums

- Following slide compares calibrated distortions for a Toy Model
  - CCoC at 15%, proportional hazard exponent 0.72, Wang shift 0.343, dual exponent 1.595, TVaR p=0.271
  - Shown compared to dual

- Next section implements this approach in detail


Premium
Loss ratio
Margin
Return on capital
VaR weight
TVaR p weight
dual
CCoC
15.00%
ph
wang
tvar
Exceedance probability
(large losses on left)
p value
(mean on left, max on right)

Simple Example / Toy Model

1. Setup and Assumptions

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Ins Co. t = 1 Cash Flows

	X1	X2	X3	X4
0	36	0	29	35
1	40	0	25	35
2	28	0	37	35
3	22	0	43	35
4	33	7	25	35
5	32	8	25	35
6	31	9	25	35
7	45	10	10	35
8	25	40	0	35
9	25	75	0	0

- Cash flows from insurer to four different counter-parties at t = 1, show all business written
- Ten equally likely scenarios, 0-9, represent all possible outcomes
- Ignore investment income, taxes, expenses etc.
- X_1 = non-cat insurance
- X_2 = cat insurance
- X_3 = equity (residual)
- X_4 = 35 xs 65 agg stop, return of collateral

What is target premium at t = 0 to pay each cash flow?

Cash Flow Summary Statistics

	X1	X2	X3	X4	total	Gross	Ceded	Net	Financing
0	36	0	29	35	100	36	0	36	64
1	40	0	25	35	100	40	0	40	60
2	28	0	37	35	100	28	0	28	72
3	22	0	43	35	100	22	0	22	78
4	33	7	25	35	100	40	0	40	60
5	32	8	25	35	100	40	0	40	60
6	31	9	25	35	100	40	0	40	60
7	45	10	10	35	100	55	0	55	45
8	25	40	0	35	100	65	0	65	35
9	25	75	0	0	100	100	35	65	0
EX	31.700	14.900	21.900	31.500	100	46.600	3.500	43.100	53.400
CV	21.5%	1.545	62.3%	33.3%	0	51.5%			32.2%
Skew	45.6%	1.791	-36.9%	-2.667	0	1.590			-78.8%

Simple Example / Toy Model

2. Natural Allocations for Dual Distortion

Algorithm for (Linear) Natural Allocation

1. Compute unit average loss grouped by total loss & sum group probabilities
2. Sort by ascending total loss (all values now distinct)
3. Compute survival function S
4. Apply distortion function g(S)
5. Difference step 4 to compute risk adjusted probabilities Q
6. Compute sum-products by unit and in total with respect to Q to obtain SRM pricing and natural allocation pricing by unit

- Step 1 replaces X_i with the conditional expectation $E[X_i \mid X]$, a random variable defined by $E[X_i \mid X](\omega) = E[X_i \mid X=X(\omega)]$
- See PIR Algorithms 11.1.1 p.271 and 15.1.1, p.397 for more detail

Spectral ask price for insurance cash flows X_1, X_2

Scenario	X1	X2	X	P	S(X)
3	22	0	22	0.1	0.9
2	28	0	28	0.1	0.8
0	36	0	36	0.1	0.7
1,4,5,6	34	6	40	0.4	0.3
7	45	10	55	0.1	0.2
8	25	40	65	0.1	0.1
9	25	75	100	0.1	0

- Collapse outcomes by value of X and sort
- $S(x) = \Pr(X > x)$

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Spectral ask price for insurance cash flows X_1, X_2

Scenario	X1	X2	X	P	S(X)	g(S)	Q=diff g(S)
3	22	0	22	0.1	0.9	0.974599	0.025401
2	28	0	28	0.1	0.8	0.923257	0.051342
0	36	0	36	0.1	0.7	0.853469	0.069788
1,4,5,6	34	6	40	0.4	0.3	0.433881	0.419588
7	45	10	55	0.1	0.2	0.299491	0.13439
8	25	40	65	0.1	0.1	0.154702	0.144789
9	25	75	100	0.1	0	0	0.154702

- Collapse outcomes by value of X and sort
- $S(x) = \Pr(X > x)$
- **Select** dual distortion $g(s) = 1 - (1 - s)^{1.59515}$
- Calibrated to 15% return with assets a = 100
- No default
- Z = Q / P

Spectral ask price for insurance cash flows X_1, X_2

Scenario	X1	X2	X	P	S(X)	g(S)	Q=diff g(S)
3	22	0	22	0.1	0.9	0.974599	0.025401
2	28	0	28	0.1	0.8	0.923257	0.051342
0	36	0	36	0.1	0.7	0.853469	0.069788
1,4,5,6	34	6	40	0.4	0.3	0.433881	0.419588
7	45	10	55	0.1	0.2	0.299491	0.13439
8	25	40	65	0.1	0.1	0.154702	0.144789
9	25	75	100	0.1	0	0	0.154702

EP	31.7	14.9	46.6
EQ	32.31	21.256	53.565
LR	0.9811	0.701	0.87

EP = loss cost
EQ = risk-loaded premium
Sum-products with P and Q columns

- Overall loss ratio is 87.0% (market assumption)
- Non-cat ask price 98.1% loss ratio (no expenses)
- Cat ask price 70.1% loss ratio

- Collapse outcomes by value of X and sort
- $S(x) = \Pr(X > x)$
- **Select** dual distortion $g(s) = 1 - (1 - s)^{1.59515}$
- Calibrated to 15% return with assets a = 100
- No default
- Z = Q / P

Spectral calculations with financing cash flows X_3, X_4

Scenario	X3	X4	Financing
3	43	35	78
2	37	35	72
0	29	35	64
1,4,5,6	25	35	60
7	10	35	45
8	0	35	35
9	0	0	0

Expected	21.9	31.5	53.4
Price	16.84935	29.58543	46.43478
Return	0.299753	0.064713	0.15

- Overall target return 15% (market)
- X_3 equity has 30% target return
- X_4 agg stop cat bond, a 6.5% return

- Descending sort order, but same Q
- Expected value of t = 1 flow (EP)
- Price is minimum acceptable bid at t = 0 for cash flows made at t = 1 (EQ)
- Price column also equals $\min_Z E[X_i Z]$
- Return = Expected value / Price – 1
- Achieves 15% overall target return
- **Implied ceded loss ratio: 64.6%**

Financing distinct from asset risk!

Simple Example / Toy Model

3. Natural Allocations for Usual Suspects

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Calibrate g to 15% return: five usual suspect distortions

	LR		
unit	X1	X2	total
distortion			
ccoc	102.8%	65.5%	87.0%
ph	101.7%	66.5%	87.0%
wang	100.1%	68.0%	87.0%
dual	98.1%	70.1%	87.0%
tvar	95.7%	72.9%	87.0%

	ROI		
unit	X3	X4	total
distortion			
ccoc	15.0%	15.0%	15.0%
ph	21.0%	11.2%	15.0%
wang	25.0%	8.9%	15.0%
dual	30.0%	6.5%	15.0%
tvar	34.9%	4.3%	15.0%

- CCoC is most sensitive to tail risk and more expensive for X_2, cat loss, and greatest benefit from reinsurance X_4
- TVaR is most sensitive to body risk (volatility) and more expensive for X1, non-cat, sees less benefit in reinsurance, and has a higher cost of equity capital X_3
- Other distortions blend between these two

Calibrate g to 15% return: five usual suspect distortions

	LR		
unit	X1	X2	total
distortion			
ccoc	102.8%	65.5%	87.0%
ph	101.7%	66.5%	87.0%
wang	100.1%	68.0%	87.0%
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tvar	95.7%	72.9%	87.0%

	ROI		
unit	X3	X4	total
distortion			
ccoc	15.0%	15.0%	15.0%
ph	21.0%	11.2%	15.0%
wang	25.0%	8.9%	15.0%
dual	30.0%	6.5%	15.0%
tvar	34.9%	4.3%	15.0%

- CCoC: negative margin for non-cat unit X_1, cat unit X_2 very expensive
- TVaR: more balanced, positive margins for both lines

- X_4 cat cover value declines with distortion body-centricity
- X_3 cost of equity increases with distortion body-centricity

Simple Example / Toy Model

4. Applications and Implications

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Application 1: Diversifying Cat Risk

- Line X_1 like a diversifying cat risk

unit	X1
distortion	
ccoc	102.8%
ph	101.7%
wang	100.1%
dual	98.1%
tvar	95.7%

- Target combined ratios by distortion vary materially
- Tail-centric distortions write at underwriting loss – see next section
- **Highlights importance of selecting distortion to match risk appetite**

A diversifying cat is a catastrophe risk from a non-peak peril, such as Chile, Australia or New Zealand.

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Application 2: Reinsurance Decision Making

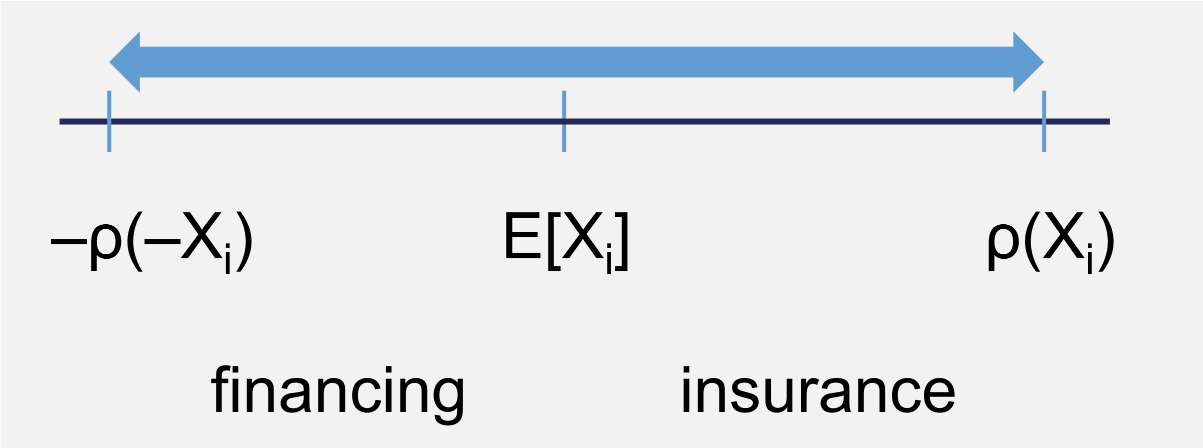
- Target cost of equity capital X_3 reflects greater aversion to earnings volatility from CCoC down to TVaR
- Break even ceded loss ratio varies from 46.0% (CCoC, tail-risk averse) to 72.9% (TVaR, volatility averse, less concerned with tail risk)
- Range of loss ratios brackets typical market pricing, showing **choice of distortion material to decision making**
- 5-point swing in net loss ratio targets

Distortion	X1	X2	X3	X4	Gross	Ceded	Net	Financing
ccoc	102.8%	65.5%	15.0%	15.0%	87.0%	46.0%	93.8%	15.0%
ph	101.7%	66.5%	21.0%	11.2%	87.0%	52.5%	91.9%	15.0%
wang	100.1%	68.0%	25.0%	8.9%	87.0%	57.5%	90.8%	15.0%
dual	98.1%	70.1%	30.0%	6.5%	87.0%	64.6%	89.5%	15.0%
tvar	95.7%	72.9%	34.9%	4.3%	87.0%	72.9%	88.4%	15.0%

Why the Natural Allocation Can Produce Negative Margins

Bounds for the Natural Allocation

- $NA(X_i)$ lies between standalone prices $\rho(X_i)$ and $-\rho(-X_i)$
- $NA(X_i) = \rho(X_i)$ if X_i is comonotonic with X
  - no diversification benefit
  - **pure insurance** risk
- $NA(X_i) = -\rho(-X_i)$ if X_i is anti-comonotonic,
  - $-X_i$ is comonotonic with X
  - **pure financing** risk



The Switcheroo: can exchange X_i for $E[X_i \mid X]$

- $E[X_i \mid X]$ is a random variable: $E[X_i \mid X](\omega) = E[X_i \mid X=X(\omega)]$
  - For simulations with distinct X values, $E[X_i \mid X] = X_i$
  - Part of algorithm for linear natural allocation

- Reduces multi-dimensional problem to one dimension, a great simplification

- Linear natural allocation to X_i and $E[X_i \mid X]$ are equal

$$NA(X_i) = E[X_i\, g'S(X)] = E[E[X_i\, g'S(X) \mid X]] = E[E[X_i \mid X]\, g'S(X)] = NA(E[X_i \mid X])$$

since $g'S(X)$ is a function of X (**linear** natural allocation)

- $\rho(E[X_i \mid X]) \le \rho(X_i)$ since $E[X_i \mid X]$ is less risky than X_i

Interpretation of Natural Allocation

- The natural allocation is the difference of two standalone premiums, each with a positive margin

- For each X_i, can write* $E[X_i \mid X] = \mathit{Ins} - \mathit{Fin}$ for two risks comonotonic with X

$$\begin{aligned} NA(X_i) &= E[X_i\, g'S(X)] = E[E[X_i \mid X]\, g'S(X)] \\ &= E[\mathit{Ins}\, g'S(X)] - E[\mathit{Fin}\, g'S(X)] = \rho(\mathit{Ins}) - \rho(\mathit{Fin}) \end{aligned}$$

  – *Ins* is the **pure insurance** part of X_i

  – *Fin* is the **pure financing** part, so-called since –Fin is anticomonotonic to X

- Natural allocation margin is the net effect of two positive margins

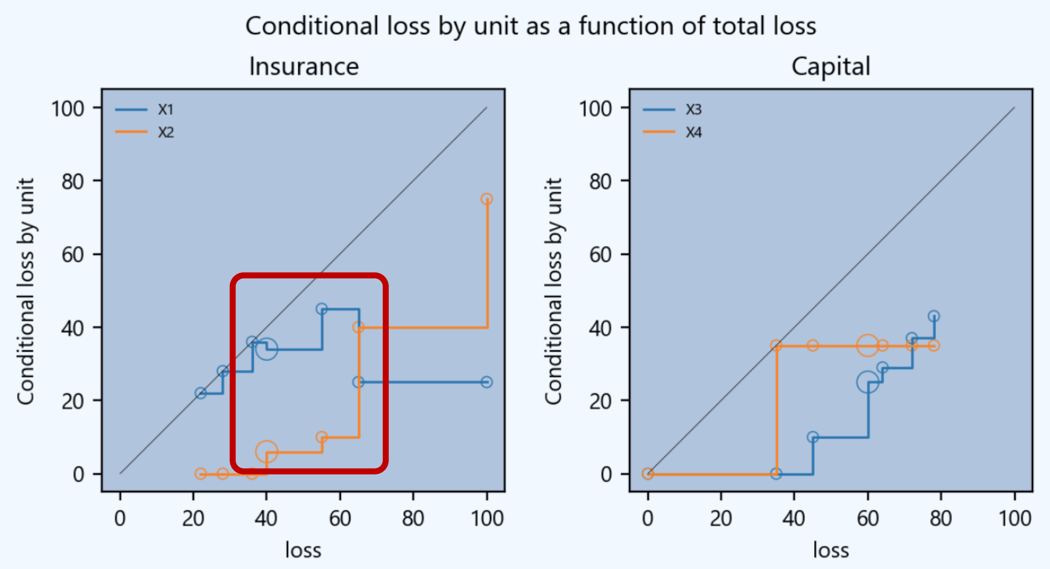
$$NA(X_i) - E[X_i] = \underbrace{(\rho(\mathit{Ins}) - E[\mathit{Ins}])}_{\text{positive insurance margin}} - \underbrace{(\rho(\mathit{Fin}) - E[\mathit{Fin}])}_{\text{financing credit}}$$

* Can *usually* write, terms and conditions apply

Interpretation of Natural Allocation

- Natural allocation margin depends on ρ's risk aversion in the insurance and financing parts of X_i
- If the financing part appears in the tail, which is usually does for a thin tail line pooled with thick, then the more tail-centric the distortion the greater the financing credit and lower the net margin
- Management may not want to credit underwriters for a capital benefit incidentally present in insurance policies
- As alternatives, could charge
  - $\rho(\mathit{Ins}) - \mathsf{E}[\mathit{Fin}]$ and ignore financing margin credit
  - $\rho(\mathsf{E}[X_i \mid X]) \ge \mathrm{NA}(X_i)$ to give credit for pooling benefit but not financing

Decomposing the Natural Allocation (dual distortion)



Left plot. Blue line shows $E[X_1 \mid X]$, which is not comonotonic with X, the diagonal. Its natural allocation has a financing credit component.

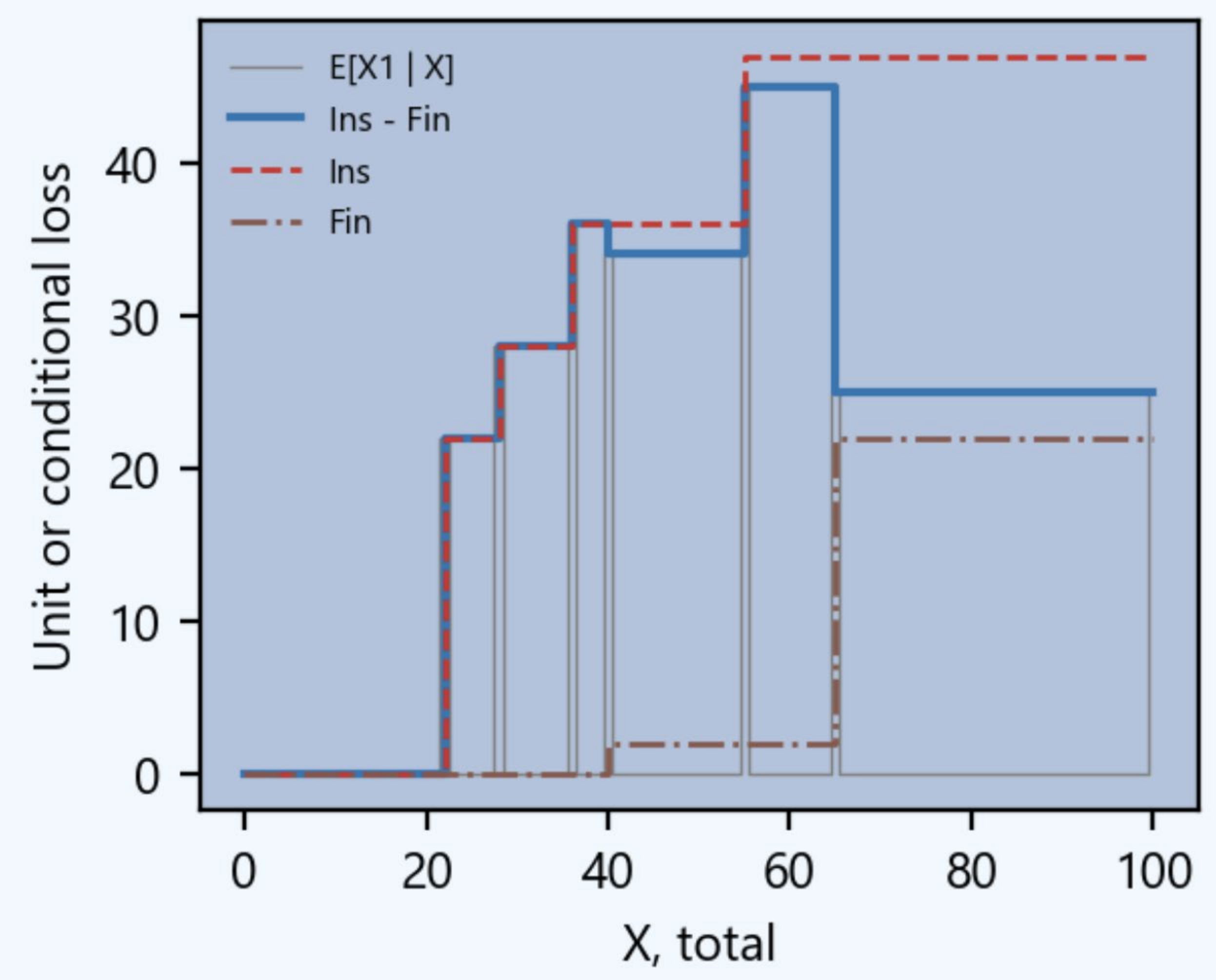
$E[X_2 \mid X]$, orange, is comonotonic with X. It is a pure insurance risk with no financing credit component.

Right plot shows the same thing for the two financing cash flows: reinsurance X4 and equity X3. Both are anticomonotonic with X meaning they are pure financing.

		NA(Xi)	ρ(Xi)	ρ(E[Xi])	ρ(Ins)	ρ(Fin)	ρ(Ins) - ρ(Fin)
unit							
X1	**EL**	31.700	31.700	31.700	37.100	5.400	31.700
X1	**Prem**	32.310	34.288	34.084	40.006	7.697	32.310

- $E[X_i \mid X]$ = *Ins* – *Fin* split into insurance and financing parts
- Insurance margin = 40.0 – 37.1 = 2.9
- Financing credit = 7.7 – 5.4 = 2.3
- Net margin 2.9 – 2.3 = 0.6
- Net NA margin = 32.3 – 31.7 = 0.6

Ins and *Fin* parts for X_1, non-cat losses



- *Ins* and *Fin* both increase with X
- Financing effect evident where *Fin* non-zero only in the tail for X ≥ 40
- Large financing loss at X=65 makes *Fin* especially valuable to tail-centric distortions

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Alternatives to the Natural Allocation

Distortion	X1	X2	X3	X4	total	Gross	Ceded	Net	Financing
ccoc	-0.874	7.839	-2.857	-4.109	0.000	6.965	4.109	2.857	-6.965
ph	-0.526	7.491	-3.803	-3.162	-0.000	6.965	3.162	3.803	-6.965
wang	-0.044	7.009	-4.378	-2.587	-0.000	6.965	2.587	4.378	-6.965
dual	0.610	6.356	-5.051	-1.915	0.000	6.965	1.915	5.051	-6.965
tvar	1.418	5.547	-5.662	-1.303	0.000	6.965	1.303	5.662	-6.965

- **Above**: margins by distortion by unit or grouping; gross and financing margins offset
- **Right**: Alternatives to the natural allocation
  - Standalone pricing, $\rho(X_i)$
  - Projected standalone, $\rho(E[X_i \mid X])$
  - Decompose into pure insurance margin (*Ins*) and pure financing credit (*Fin*) and omit financing credit
  - X_2 is pure insurance, so *Fin* margin = 0

Distortion	Unit	NA(Xi)	ρ(Xi)	ρ(E[Xi | X])	ρ(Ins)	ρ(Fin)	ρ(Ins) - ρ(Fin)
ccoc	X1	-0.874	1.735	1.735	1.291	2.165	-0.874
	X2	7.839	7.839	7.839	7.839	0.000	7.839
ph	X1	-0.526	2.067	1.894	1.894	2.419	-0.526
	X2	7.491	7.559	7.491	7.491	0.000	7.491
wang	X1	-0.044	2.297	2.094	2.349	2.393	-0.044
	X2	7.009	7.123	7.009	7.009	0.000	7.009
dual	X1	0.610	2.588	2.384	2.906	2.297	0.610
	X2	6.356	6.536	6.356	6.356	0.000	6.356
tvar	X1	1.418	2.906	2.906	3.429	2.010	1.418
	X2	5.547	5.547	5.547	5.547	0.000	5.547

Contact Information & Resources

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Contact Information and Resources

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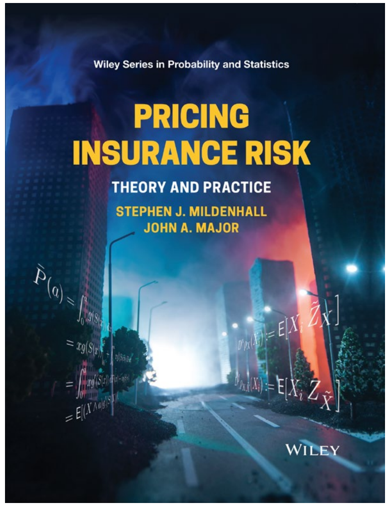
Biography

Stephen Mildenhall is an FCAS with a distinguished 30-year career in insurance and academia. He leads Analytics at QualRisk, focusing on risk and capital optimization in insurance and financial services. Previously, he was an Assistant Professor at St. John's University, New York, and held leadership positions at Aon, including Global CEO of Analytics and head of Aon Reinsurance Analytics. His career began in pricing at Kemper Insurance and CNA, focusing on personal, commercial, and reinsurance lines. At QualRisk, he continues to engage in bespoke consulting projects, while also programming the `aggregate` Python package and contributing to the literature in his field.



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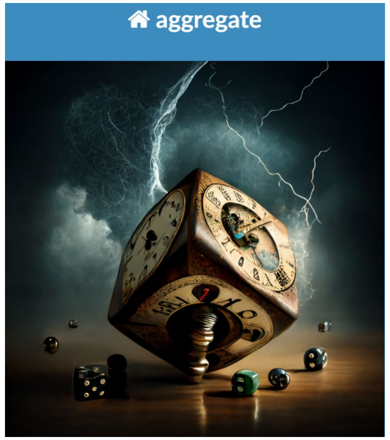
- Case study exhibits
- Supplemental exhibits
- Presentations
- Errata



Software documentation

https://aggregate.readthedocs.io/en/latest/

Code

https://www.github.com/mynl.aggregate

Appendix:

Details of Reinsurance Cash Flows

Explanation of Reinsurance Cash Flow

Cash flow for	Time t = 0 (fixed)	Time t = 1 (variable)
Insurance policy	In: premium	Out: losses
Bond	In: principal	Out: repayment & interest
Equity	In: capital paid-in	Out: entity residual value
Reinsurance	**Out**: ceded premium	**In**: ceded losses
Alternative reinsurance	In: purchase return of collateral	Out: return of collateral

Want cash flows In / Out not Out / In

- Reinsurers quote premium P required to pay ceded losses L
- For a collateralized 35 xs 65 layer
  - Ceded premium P ↔ pay 35 – P for return of collateral
  - Ceded loss L ↔ return 35 – L
  - Equivalent to standard reinsurance plus a risk-free loan of 35 from the reinsurer to the insurer at t=0
  - Compare cat bonds