Pricing Insurance Risk

Module M: Severe Convective Storm and Hurricane Model Example

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convex risk

independent | informed | imaginative

This presentation expresses my personal opinion.

The ideas and concepts presented are the result of joint work with John Major. Look for our coming book *Pricing Insurance Risk: A Guide for the Working Actuary* **to be published by John Wiley and Sons, Inc.**

Pricing Insurance Risk Course
https://www.convexrisk.com/pirc

Hurricane and Severe Convective Storm Example

Overview

- HU: extreme US hurricane exposure
- SCS: Midwest tornado hail exposure
- Both severities limited at 1.2B for numerical stability

`aggregate` program

```
port Variance_Example
    agg SCS 70 claims 1200000000 x 0 sev 1635.984429995927 * lognorm 1.9 poisson
    agg HU 2 claims 1200000000 x 0 sev 6002.912217261018 * lognorm 2.5 poisson
```

Hurricane and Severe Convective Storm Example

statistic	HU	SCS	total
Frequency	2	70.00	72.00
Severity	136,079	9,947	13,450
Modeled Mean	270,433	689,980	960,411
Modeled CV	10.69	0.733	3.05
Modeled Skew	110.8	24.45	106.0
Modeled Kurt	22,381	7,655	21,057

Table 1: Portfolio statistics based on FFT computation. Acutal severies are are 9,946.7 and 136,625.7. Actual expected losses are 696,271.7 and 273,251.3.

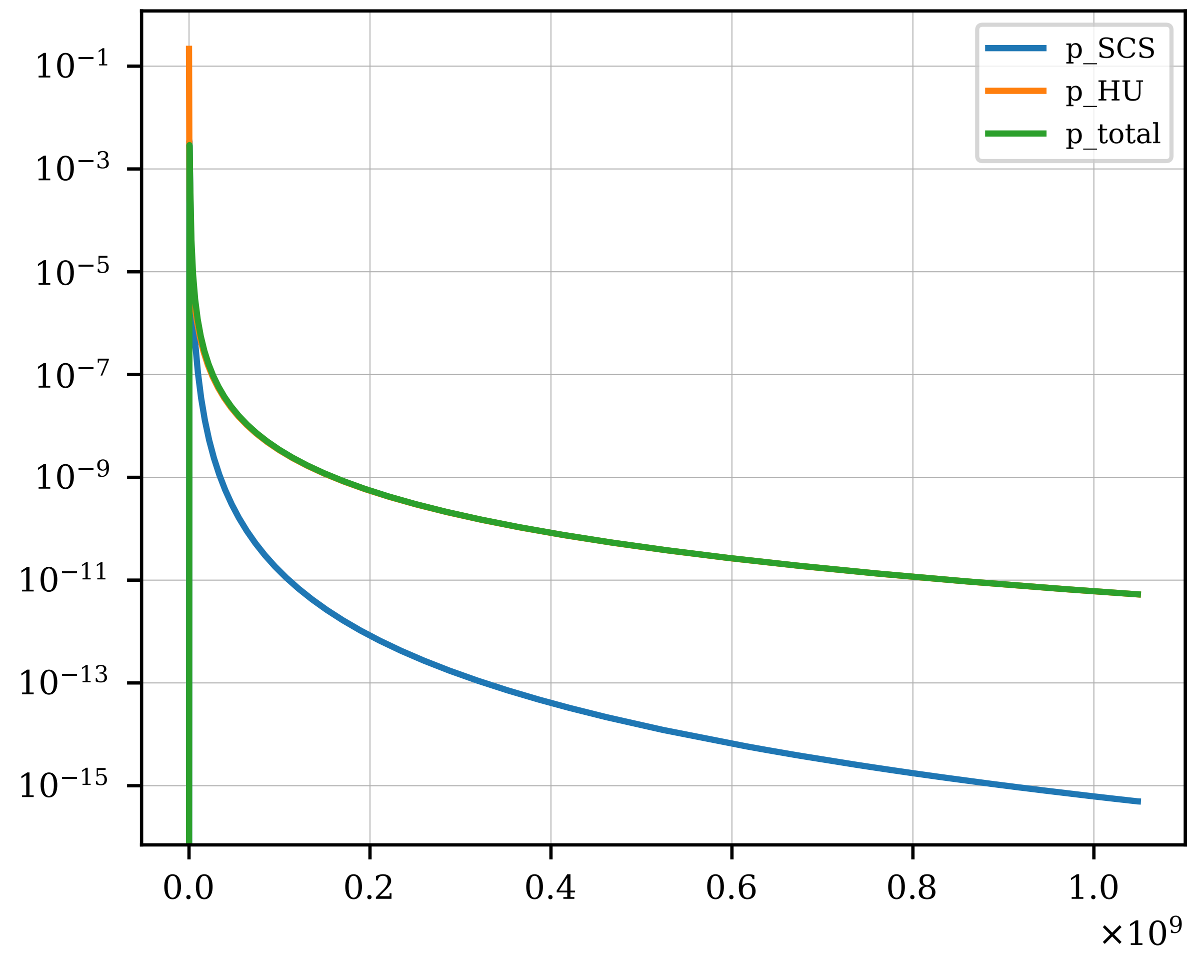

Figure 1: Log density for each line and in total.

Capital and Pricing Assumptions

- Capital standard 0.99-TVaR, estimated implied assets 14,664,000
  - Substantially above 0.99-VaR of 5,094,000
  - Corresponds to 0.998-VaR
- Limited expected value of total losses 914,180 vs. modeled unlimited loss 960,411
  - Expected policyholder deficit 0.048
  - HU expected loss 225,304, allowing for default
  - SCS expected 688,876
- Target return on capital 0.05
- Total technical premium $1{,}568{,}933 = 0.952 \times 914{,}180 + 0.048 \times 14{,}664{,}000$
- Technical loss ratio 0.583

Bivariate Density and Scatter Plots

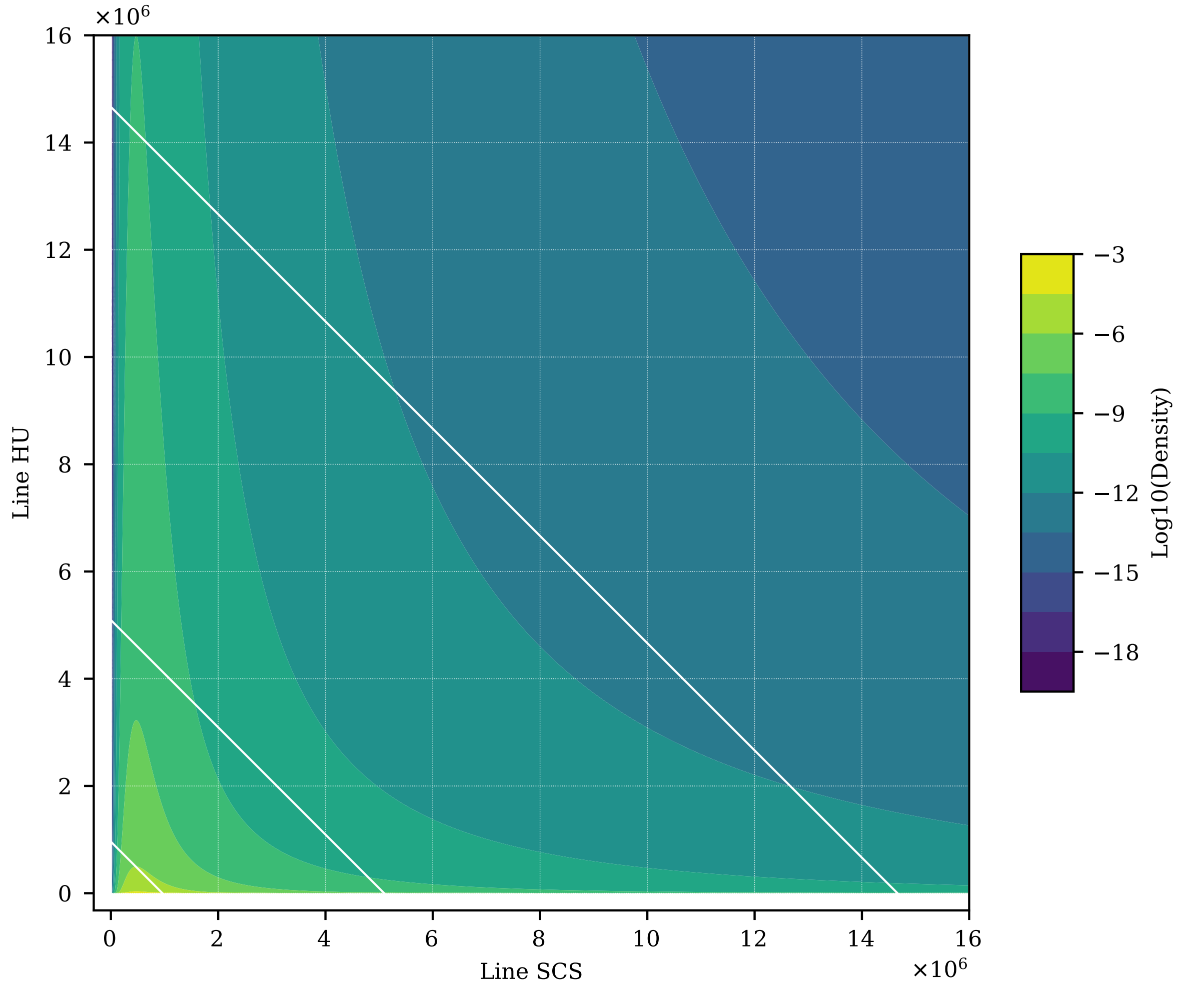

Figure 2: Bivariate log density. Diagonal lines at EL {el:,.0f}, {reg_p}-VaR {a0:,.f}, and capital level {a:,.0f}.

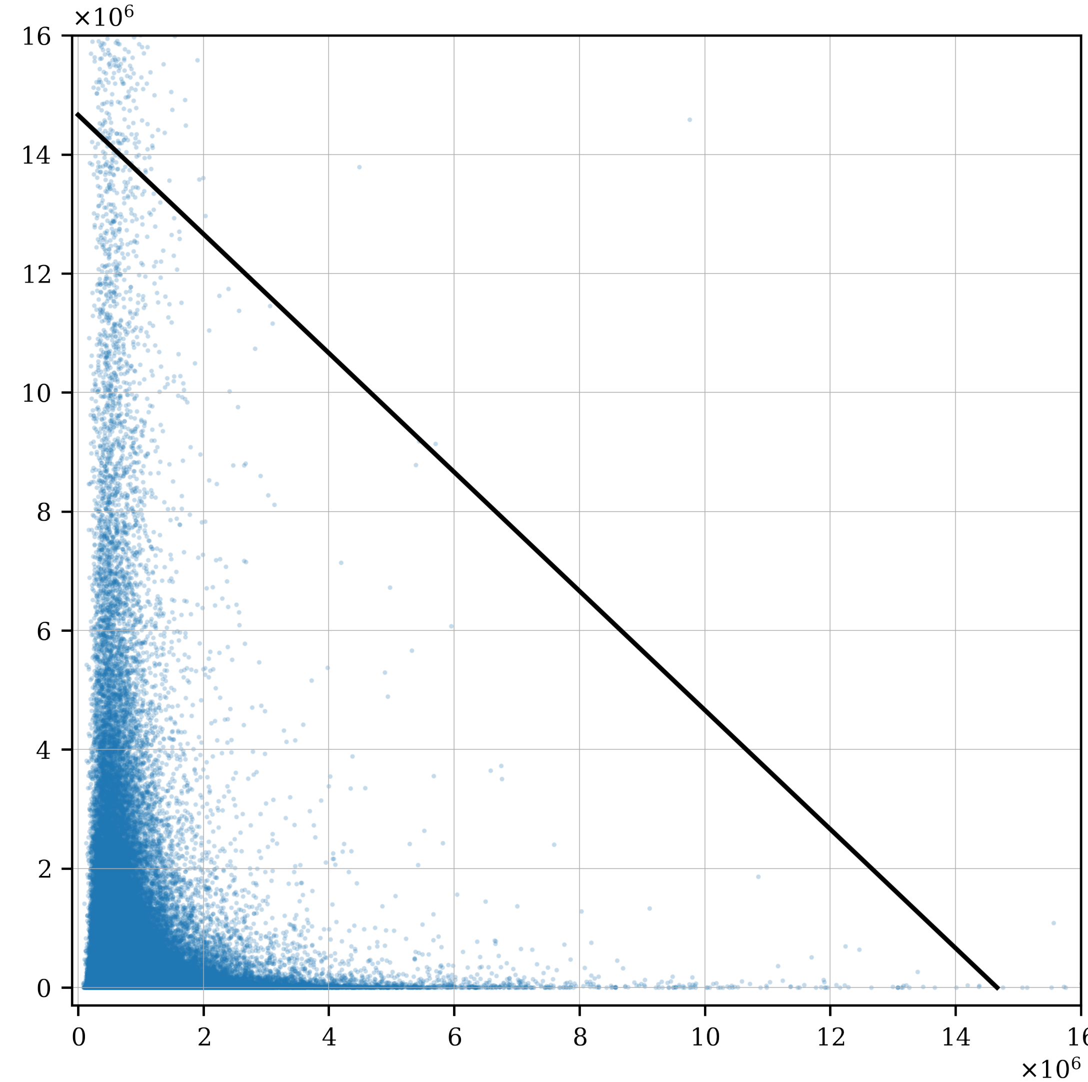

Figure 3: Bivariate density: 500000 simulated outcomes. Diagonal line shows 14,664,000 capital constraint.

Pricing Statistics by Method

Method	Premium			LR			Margin			Allocated Capital			ROE		
	HU	SCS	total	HU	SCS	total	HU	SCS	total	HU	SCS	total	HU	SCS	total
EL	386,672	1.182M	1.569M	0.583	0.583	0.583	161,367	493,386	654,753	3.227M	9.868M	13.095M	0.05	0.05	0.05
Dist tvar	549,737	1.019M	1.569M	0.41	0.676	0.583	324,433	330,320	654,753	10.787M	2.308M	13.095M	0.0301	0.143	0.05
Dist wang	677,398	891,536	1.569M	0.333	0.773	0.583	452,093	202,660	654,753	11.006M	2.089M	13.095M	0.0411	0.097	0.05
EqRiskVaR	735,528	833,405	1.569M	0.306	0.827	0.583	510,224	144,530	654,753	10.204M	2.891M	13.095M	0.05	0.05	0.05
Dist ph	743,521	825,412	1.569M	0.303	0.835	0.583	518,217	136,536	654,753	11.306M	1.789M	13.095M	0.0458	0.0763	0.05
EqRiskTVaR	749,022	819,911	1.569M	0.301	0.84	0.583	523,718	131,036	654,753	10.474M	2.621M	13.095M	0.05	0.05	0.05
ScaledVaR	749,855	819,079	1.569M	0.3	0.841	0.583	524,550	130,203	654,753	10.491M	2.604M	13.095M	0.05	0.05	0.05
ScaledTVaR	760,275	808,658	1.569M	0.296	0.852	0.583	534,971	119,783	654,753	10.699M	2.396M	13.095M	0.05	0.05	0.05
MerPer	770,004	1.027M	1.797M	0.293	0.671	0.509	544,700	338,054	882,753	10.894M	6.761M	17.655M	0.05	0.05	0.05
coTVaR	835,759	733,422	1.569M	0.27	0.939	0.583	610,455	44,547	654,992	12.209M	890,934	13.100M	0.05	0.05	0.05
TVaR	860,294	836,626	1.697M	0.262	0.823	0.539	634,990	147,750	782,740	12.700M	2.955M	15.655M	0.05	0.05	0.05
covar	860,739	708,347	1.569M	0.262	0.973	0.583	635,434	19,471	654,753	12.709M	389,429	13.095M	0.05	0.05	0.05
VaR	862,576	853,405	1.716M	0.261	0.807	0.533	637,271	164,530	801,801	12.745M	3.291M	16.036M	0.05	0.05	0.05
EqRiskEPD	876,852	692,082	1.569M	0.257	0.995	0.583	651,547	3,206	654,753	13.031M	64,121	13.095M	0.05	0.05	0.05
ScaledEPD	899,810	669,124	1.569M	0.25	1.03	0.583	674,505	-19,752	654,753	13.490M	-395,037	13.095M	0.05	0.05	0.05
Dist roe	971,529	597,405	1.569M	0.232	1.15	0.583	746,224	-91,471	654,753	14.921M	-1.826M	13.095M	0.05	0.0501	0.05
EPD	3.801M	724,386	4.526M	0.0593	0.951	0.202	3.576M	35,510	3.611M	71.516M	710,198	72.226M	0.05	0.05	0.05

Table 2: Methods sorted by HU premium. Dist methods use a DRM. MerPer, TVaR, VaR and EPD methods are not additive. All classical methods produce the same ROE for each line, whereas modern DRM methods differentiate between lines, other than the constant ROE distortion (up to rounding). TVaR distortion uses 0.639-TVaR, which replicates total premium. coTVaR is the classical asset-based TVaR allocation. As is commonly the case, TVaR distortion provides the least differentiated pricing, being very close to EL and constant ROE distortion produces the most differentiated. Distortion returns to allocated capital are lower for higher-risk HU because it consumes more, cheaper high return period capital and less, more expensive equity capital. The EL premium cannot be obtained using a DRM.

Loss and Premium Functions

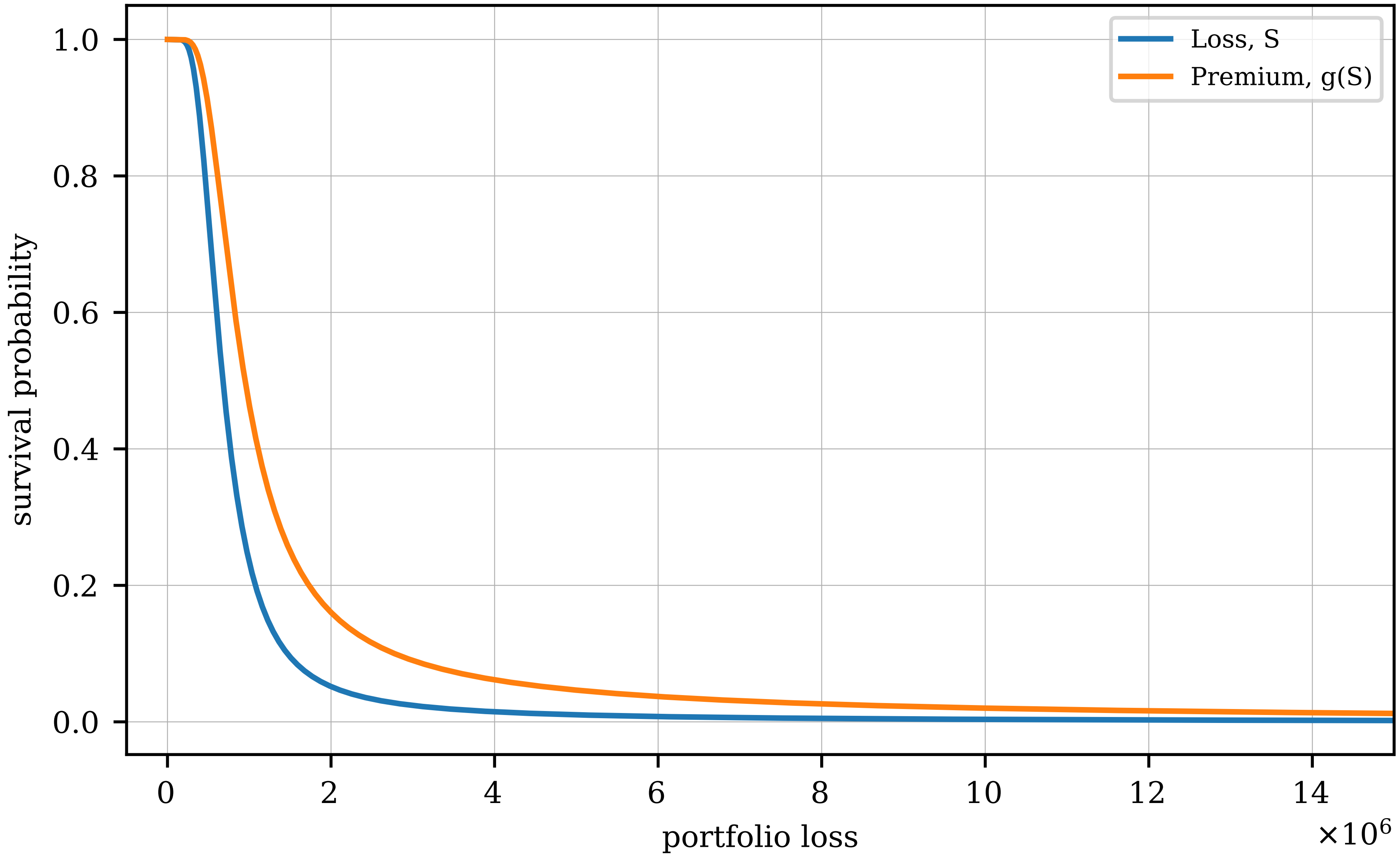

Figure 4: Loss survival function S, which is the layer loss density, and premium loss density $g(S)$ for the Wang distortion.

Alpha, Beta and Kappa by Line

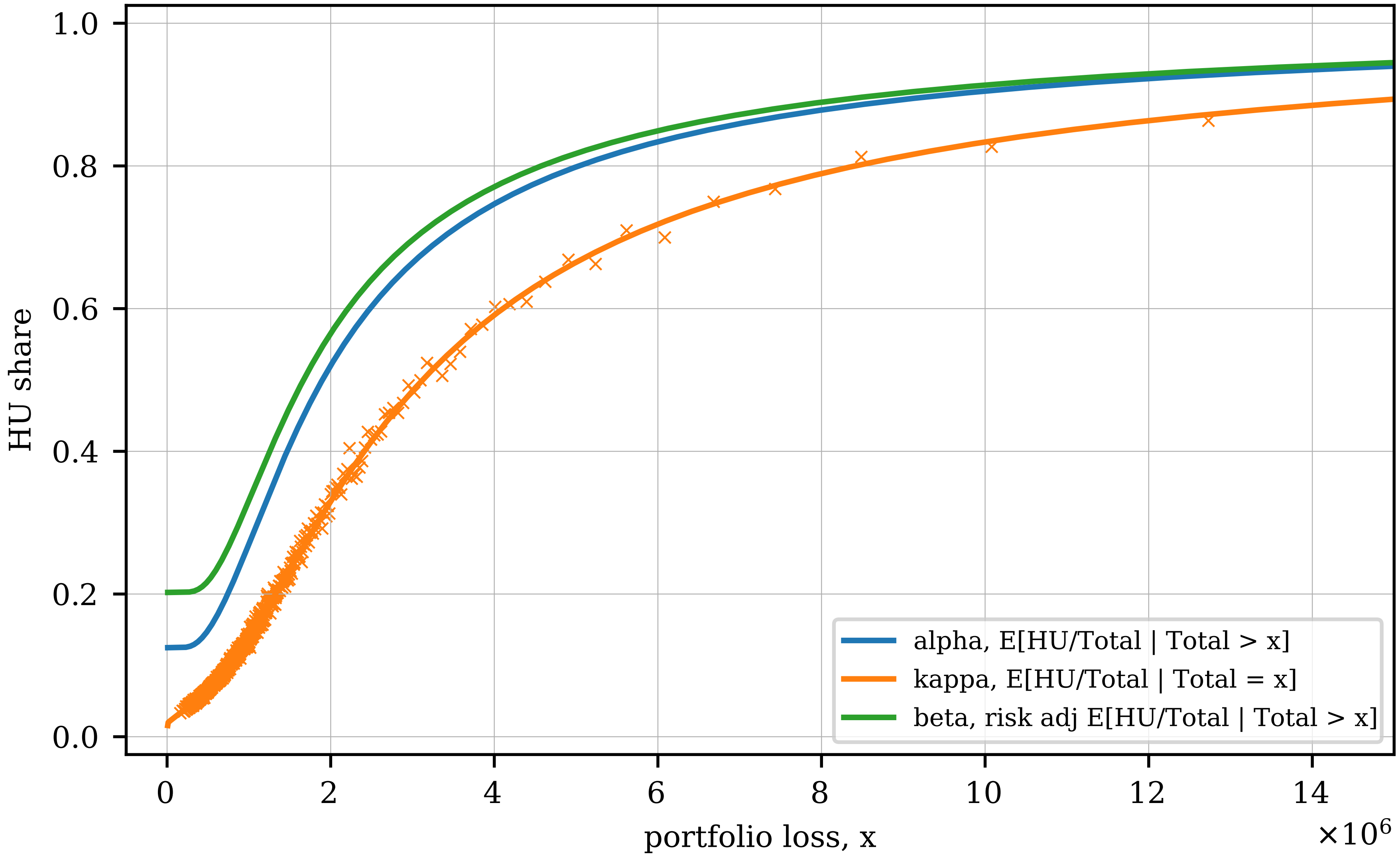

Figure 5: Relative risk functions, α, β and κ by line. Includes estimated κ based on 500,000 simulations, bucketed into 1000 buckets with 500 obervations each. The β function uses Wang distortion, with shape parameter 0.635. Since κ is increasing α lies above κ and since α is increasing β lies above α.

Margin Density Functions

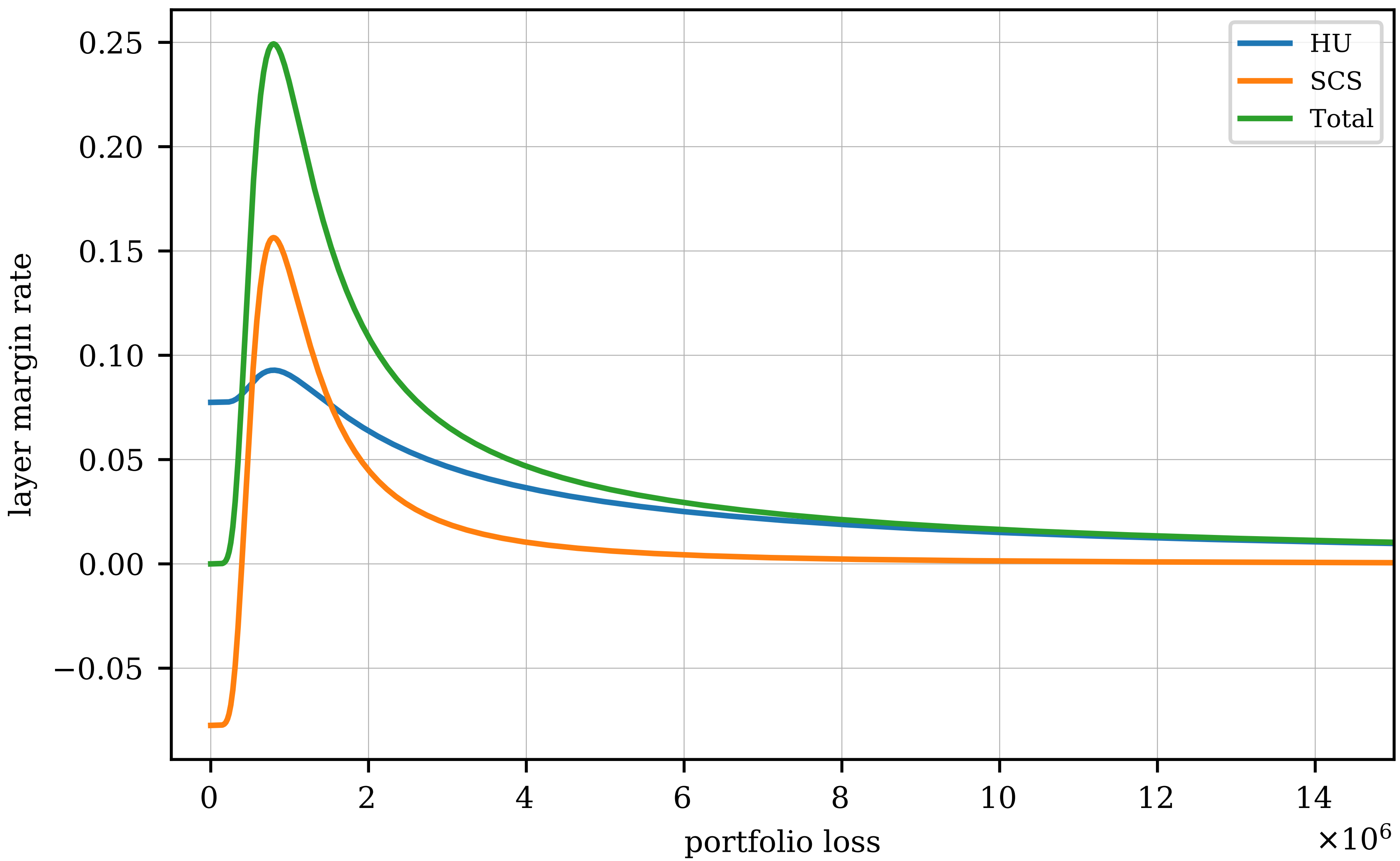

Figure 6: The margin density $\beta_i(x)g(S(x)) - \alpha_i(x)S(x)$ for the Wang distortion. The SCS negative margin for small total loss amounts represents a transfer from HU to SCS to compensate for the benefit HU receives from pooling. The total margin, $g(S(x)) - S(x) \geq 0$ for all x.

Capital Density Functions

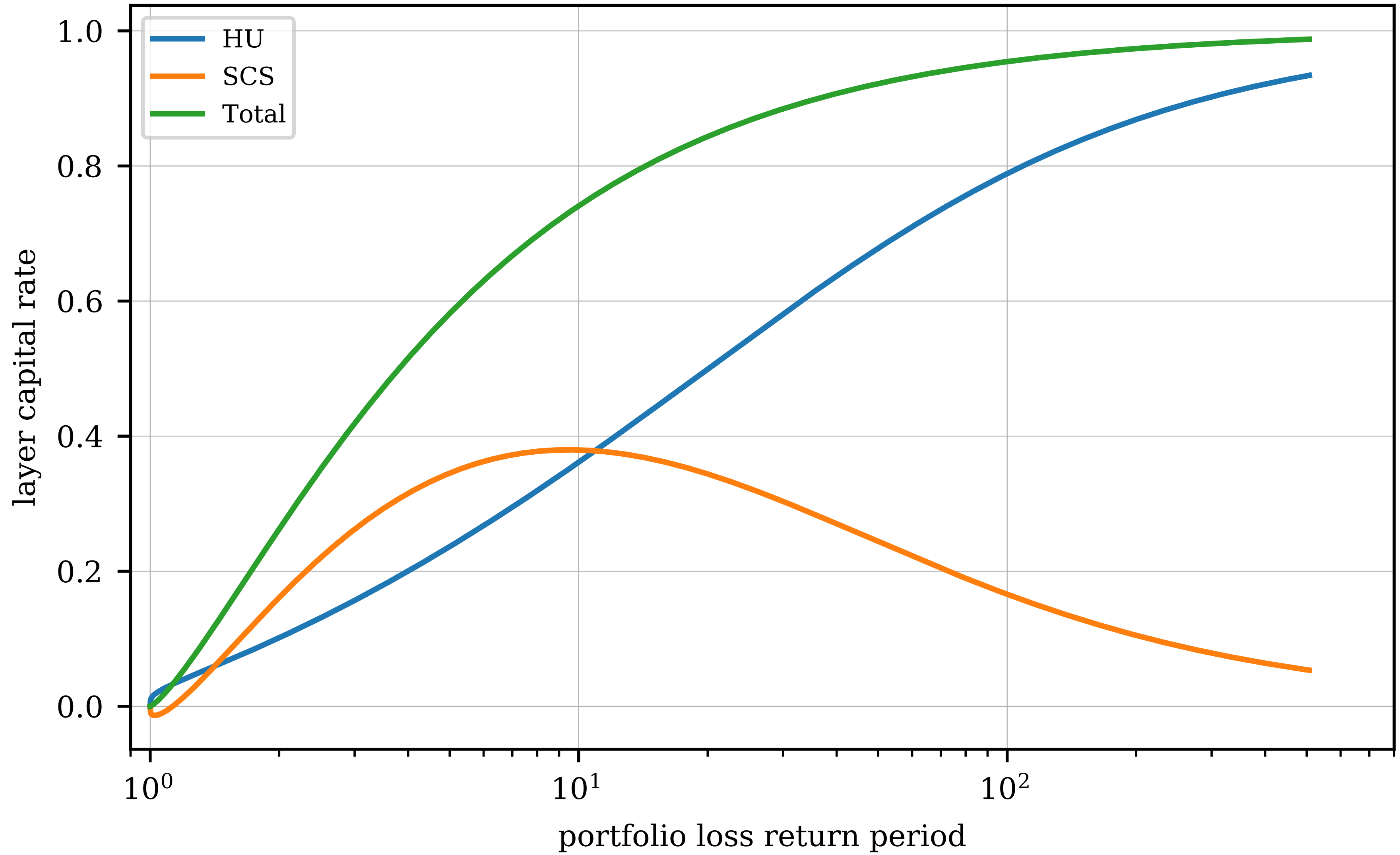

Figure 7: The capital density for the Wang distortion. Lower layers are funded almost entirely by premium. Upper layers are funded by equity. Upper layer assets accrue predominantly to the benefit of HU because it has a much thicker tail. The SCS negative capital density for small total loss amounts is an artefact of the negative margin. For such low layers, assets are fully funded by premiums and there is no net equity.