

Pricing Insurance Risk

Module G: Comparative Pricing Across Different Methods and Lines

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G.01. Stand-Alone by Line and Portfolio Pricing

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Stand-Alone by Line and Portfolio Pricing

Stand-Alone By Line Pricing

- Method applied to each line on a stand-alone basis
- Premium determined by amount of assets a and cost of capital r
- $P = L + r(a - P) = vL + da$, where $v = 1/(1 + r)$, $d = r/(1 + r)$, and expected loss L

Amount of Capital

- VaR at 0.996 level, total and by line
- Distortion pricing based on VaR 0.996 capital standard

No Allocation

- Methods in this section do not require asset allocation to line
- Prefix sa indicates a method applied on a stand-alone basis

Stand-Alone by Line and Portfolio Pricing for Constant ROE, Bagged Bond Base Distortion, and Traditional Methods

Cost of Capital

- Distortion methods determine cost of capital by asset layer; portfolio cost emerges based on capital consumed at each probability level
- Standard methods assume a constant cost of capital, set equal to $r = 0.141$, the ROE determined by the base distortion on the gross portfolio
- traditional - no default pricing uses unlimited expected losses, $E[X_i]$, not allowing for default
- traditional pricing uses expected recovered losses $E[X_i \wedge a]$, allowing for potential default

Distortions

- Bagged Bond Base
- Gross ROE at $r = 0.141$ gross ROE, will replicate traditional on a stand-alone basis

Gross Stand-Alone Traditional and Distortion

stat	method	BOP	CAuto	PAuto	Property	SCS	Wind	sop	total
Expected Loss	traditional - no default	20,000	10,000	40,000	20,000	4,997	5,000	99,997	99,997
	traditional	19,985	9,995	39,994	19,988	4,901	4,175	99,039	99,175
	sa gross roe	19,985	9,995	39,994	19,988	4,901	4,175	99,039	99,175
	sa 1000 resamples, 0.5 of data	19,985	9,995	39,994	19,988	4,901	4,175	99,039	99,175
Loss Ratio	traditional - no default	0.873	0.911	0.966	0.892	0.461	0.208	0.754	0.839
	traditional	0.873	0.911	0.966	0.892	0.456	0.179	0.752	0.837
	sa gross roe	0.873	0.911	0.966	0.892	0.456	0.179	0.752	0.837
	sa 1000 resamples, 0.5 of data	0.855	0.897	0.96	0.877	0.445	0.182	0.745	0.837
Margin	traditional - no default	2,897	976.0	1,408	2,409	5,831	19,074	32,596	19,254
	traditional	2,899	976.5	1,409	2,410	5,843	19,176	32,714	19,355
	sa gross roe	2,899	976.5	1,409	2,410	5,843	19,176	32,714	19,355
	sa 1000 resamples, 0.5 of data	3,378	1,142	1,663	2,816	6,107	18,723	33,829	19,355
Premium	traditional - no default	22,897	10,976	41,408	22,409	10,828	24,074	132,593	119,251
	traditional	22,884	10,972	41,403	22,398	10,744	23,352	131,753	118,530
	sa gross roe	22,884	10,972	41,403	22,398	10,744	23,352	131,753	118,530
	sa 1000 resamples, 0.5 of data	23,363	11,137	41,657	22,804	11,008	22,899	132,868	118,530
Leverage	traditional - no default	1.11	1.59	4.14	1.31	0.262	0.178	0.573	0.873
	traditional	1.11	1.58	4.14	1.31	0.259	0.172	0.568	0.863
	sa gross roe	1.11	1.58	4.14	1.31	0.259	0.172	0.568	0.863
	sa 1000 resamples, 0.5 of data	1.16	1.65	4.28	1.37	0.267	0.168	0.575	0.863
Surplus	traditional - no default	20,553	6,924	9,992	17,091	41,372	135,326	231,257	136,599
	traditional	20,566	6,928	9,997	17,102	41,456	136,048	232,097	137,320
	sa gross roe	20,566	6,928	9,997	17,102	41,456	136,048	232,097	137,320
	sa 1000 resamples, 0.5 of data	20,087	6,763	9,743	16,696	41,192	136,501	230,982	137,320
ROE	traditional - no default	0.141	0.141	0.141	0.141	0.141	0.141	0.141	0.141
	traditional	0.141	0.141	0.141	0.141	0.141	0.141	0.141	0.141
	sa gross roe	0.141	0.141	0.141	0.141	0.141	0.141	0.141	0.141
	sa 1000 resamples, 0.5 of data	0.168	0.169	0.171	0.169	0.148	0.137	0.146	0.141
Assets	traditional - no default	43,450	17,900	51,400	39,500	52,200	159,400	363,850	255,850
	traditional	43,450	17,900	51,400	39,500	52,200	159,400	363,850	255,850
	sa gross roe	43,450	17,900	51,400	39,500	52,200	159,400	363,850	255,850
	sa 1000 resamples, 0.5 of data	43,450	17,900	51,400	39,500	52,200	159,400	363,850	255,850

Gross Stand-Alone Traditional and Distortion

Data elements

- traditional - no default premium $P_i = vE[X_i] + da$
- traditional premium $P_i = vE[X_i \wedge a] + da$
- sa base is the base convex distortion applied on a stand-alone basis
- gross roe has equal ROE to the base, applied on a stand-alone basis
- sop = sum of parts across individual lines
- total = portfolio considered as a whole

Gross Stand-Alone Traditional and Distortion

Loss Ratio and ROE by line using stand-alone distortion pricing

- Loss ratios in reasonable range and aligned with volatility by line
- sa base ROEs **vary by line**—despite applying the **same** distortion on a stand-alone basis
 - Each line consumes a unique mix of capital at different return periods, with different costs
 - Line mix weights down to a unique ROE
- Counter-intuitively **more volatile** lines get a **lower ROE** because their thin tails consume relatively **less cheap** tail capital
- Differences in ROE by line more pronounced for heterogeneous gross book than net, shown on next slide

Net Traditional and Distortion Stand-Alone

stat	method	BOP	CAuto	PAuto	Property	SCS	Wind	sop	total
Expected Loss	traditional - no default	20,000	10,000	40,000	20,000	4,741	2,694	97,435	97,435
	traditional	19,985	9,995	39,994	19,988	4,720	2,659	97,341	97,405
	sa net roe	19,985	9,995	39,994	19,988	4,720	2,659	97,341	97,405
	sa 1000 resamples, 0.5 of data	19,985	9,995	39,994	19,988	4,720	2,659	97,341	97,405
Loss Ratio	traditional - no default	0.856	0.898	0.961	0.877	0.55	0.386	0.851	0.937
	traditional	0.856	0.898	0.961	0.877	0.549	0.383	0.85	0.937
	sa net roe	0.856	0.898	0.961	0.877	0.549	0.383	0.85	0.937
	sa 1000 resamples, 0.5 of data	0.855	0.897	0.96	0.877	0.547	0.383	0.85	0.937
Margin	traditional - no default	3,370	1,135	1,638	2,803	3,882	4,284	17,112	6,549
	traditional	3,372	1,136	1,639	2,804	3,885	4,289	17,126	6,553
	sa net roe	3,372	1,136	1,639	2,804	3,885	4,289	17,126	6,553
	sa 1000 resamples, 0.5 of data	3,378	1,142	1,663	2,816	3,911	4,277	17,188	6,553
Premium	traditional - no default	23,370	11,135	41,638	22,803	8,623	6,978	114,548	103,984
	traditional	23,357	11,131	41,633	22,792	8,605	6,948	114,467	103,958
	sa net roe	23,357	11,131	41,633	22,792	8,605	6,948	114,467	103,958
	sa 1000 resamples, 0.5 of data	23,363	11,137	41,657	22,804	8,631	6,937	114,529	103,958
Leverage	traditional - no default	1.16	1.65	4.27	1.37	0.373	0.273	1.12	2.67
	traditional	1.16	1.64	4.26	1.36	0.372	0.272	1.12	2.66
	sa net roe	1.16	1.64	4.26	1.36	0.372	0.272	1.12	2.66
	sa 1000 resamples, 0.5 of data	1.16	1.65	4.28	1.37	0.373	0.271	1.12	2.66
Surplus	traditional - no default	20,080	6,765	9,762	16,697	23,127	25,522	101,952	39,016
	traditional	20,093	6,769	9,767	16,708	23,145	25,552	102,033	39,042
	sa net roe	20,093	6,769	9,767	16,708	23,145	25,552	102,033	39,042
	sa 1000 resamples, 0.5 of data	20,087	6,763	9,743	16,696	23,119	25,563	101,971	39,042
ROE	traditional - no default	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.168
	traditional	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.168
	sa net roe	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.168
	sa 1000 resamples, 0.5 of data	0.168	0.169	0.171	0.169	0.169	0.167	0.169	0.168
Assets	traditional - no default	43,450	17,900	51,400	39,500	31,750	32,500	216,500	143,000
	traditional	43,450	17,900	51,400	39,500	31,750	32,500	216,500	143,000
	sa net roe	43,450	17,900	51,400	39,500	31,750	32,500	216,500	143,000
	sa 1000 resamples, 0.5 of data	43,450	17,900	51,400	39,500	31,750	32,500	216,500	143,000

Net Traditional and Distortion Stand Alone

- Same distortions as gross
- Regulatory assets decrease 44% from 255,850 to 143,000
- ROE increases from 0.141 to 0.168, because use of lower-ROE, higher layer tail asset decreases
- Very tight range for target ROEs across lines

Gross Minus Net Statistics

stat	method	BOP	CAuto	PAuto	Property	SCS	Wind	sop	total
Expected Loss	traditional - no default					255	2,306	2,561	2,561
	traditional					181	1,516	1,697	1,769
	roe					181	1,516	1,697	1,769
	sa 1000 resamples, 0.5 of data					181	1,516	1,697	1,769
Loss Ratio	traditional - no default	0.018	0.013	0.005	0.015	-0.088	-0.178	-0.096	-0.098
	traditional	0.018	0.013	0.005	0.015	-0.092	-0.204	-0.099	-0.100
	roe	0.018	0.013	0.005	0.015	-0.092	-0.204	-0.099	-0.100
	sa 1000 resamples, 0.5 of data					-0.102	-0.201	-0.105	-0.100
Margin	traditional - no default	-473	-159	-230	-394	1,950	14,791	15,484	12,705
	traditional	-474	-160	-230	-394	1,958	14,887	15,589	12,802
	roe	-474	-160	-230	-394	1,958	14,887	15,589	12,802
	sa 1000 resamples, 0.5 of data					2,196	14,446	16,642	12,802
Premium	traditional - no default	-473	-159	-230	-394	2,205	17,097	18,045	15,267
	traditional	-474	-160	-230	-394	2,140	16,403	17,286	14,572
	roe	-474	-160	-230	-394	2,140	16,403	17,286	14,572
	sa 1000 resamples, 0.5 of data					2,377	15,962	18,339	14,572
Leverage	traditional - no default	-0.050	-0.061	-0.121	-0.054	-0.111	-0.095	-0.550	-1.792
	traditional	-0.050	-0.061	-0.121	-0.054	-0.113	-0.100	-0.554	-1.800
	roe	-0.050	-0.061	-0.121	-0.054	-0.113	-0.100	-0.554	-1.800
	sa 1000 resamples, 0.5 of data					-0.106	-0.104	-0.548	-1.800
Surplus	traditional - no default	473	159	230	394	18,245	109,803	129,305	97,583
	traditional	474	160	230	394	18,310	110,497	130,064	98,278
	roe	474	160	230	394	18,310	110,497	130,064	98,278
	sa 1000 resamples, 0.5 of data					18,073	110,938	129,011	98,278
ROE	traditional - no default	-0.027	-0.027	-0.027	-0.027	-0.027	-0.027	-0.027	-0.027
	traditional	-0.027	-0.027	-0.027	-0.027	-0.027	-0.027	-0.027	-0.027
	roe	-0.027	-0.027	-0.027	-0.027	-0.027	-0.027	-0.027	-0.027
	sa 1000 resamples, 0.5 of data					-0.021	-0.030	-0.022	-0.027
Assets	traditional - no default					20,450	126,900	147,350	112,850
	traditional					20,450	126,900	147,350	112,850
	roe					20,450	126,900	147,350	112,850
	sa 1000 resamples, 0.5 of data					20,450	126,900	147,350	112,850

Gross Minus Net Statistics

- Loss differences reflect cessions to cat for SCS and Wind program
- Impact to non-reinsured lines caused by different gross and net total ROEs: traditional methods separately calibrated to different ROEs
- Substantial reduction in SCS and Wind premium
- Traditional premium and margins increase (loss ratio decreases) for non-cat lines because ROE target increases
- Distortion unchanged, and so distortion premium unchanged, for unreinsured non-cat lines

Concluding Comments

Gross minus net does not equal ceded

- Gross minus net shows the difference in **economic premium** for the two portfolios caused by a cession
- The cost of reinsurance on SCS and Wind is not quantified or germane
- An efficient reinsurance program should cost less than the difference in economic value

Market premiums support optimal capital structure

- Market premiums reflect the economics of bearing risk against an optimal capital base, blending equity, debt, and reinsurance
- Market premiums may produce an inadequate ROE if risk assumed against a sub-optimal capital structure

G.02. Gross and Net Line-Level and Portfolio Pricing

Roadmap: Multiline Pricing

Analysis proceeds in three steps

In this section:

1. Stand-alone traditional and distortion methods vs. natural allocation for base and constant roe distortions
 - Gross by line
 - Net by line
 - Explain gross to net transition for the natural allocation, base convex distortion
 - This section
2. Natural allocation by line for other cat bond calibrated distortions

In the next section:

3. Distortion vs. traditional methods by line pricing calibrated to consistent total return

Multiline Pricing

Allocation

- Previous section considered stand-alone pricing
- This section considers pricing by line within a multiline portfolio
- Multiline pricing requires an allocation of
 - premium or
 - margin or
 - capital and an assumption about cost of capital
- Begin by considering distortion approaches to allocation
- Other Traditional approaches to allocation considered in the next section

Multiline Pricing

Losses

- **For the remainder of the presentation expected losses always account for possible default**
- Expected losses become a function of available assets

Multiline Pricing

Distortion methods

- base uses the bootstrapped US Wind distortion
- gross roe distortion uses constant gross ROE
- Distortion methods prefixed Dist
- Distortion methods have a unique **natural allocation** of premium and margin by line, described on the next slide

The Natural Allocation

The natural allocation is a conditional measure approach to premium allocation that applies to distortion methods

- It is consistent with marginal allocations in the absence of default
- Example: coTVaR
- It assumes limited liability with equal priority in default
- Applied to the identity (risk neutral) distortion $g(s) = s$ it computes the expected value of recoveries to line i
- Applied to a non-identity distortion g it allocates the premium determined by ρ_g
- Allocated premium and loss costs determine allocated margin
- Allocated capital by layer equals margin divided by layer ROE $((g(s) - s)/(1 - g(s)))$, which is equal across all lines by law invariance
- See Major and Mildenhall (2020) <https://arxiv.org/pdf/2008.12427.pdf> for more details

Gross Traditional Stand-Alone vs Natural Allocation Margin by Line

method	BOP	CAuto	PAuto	Property	SCS	Wind	sop	total
traditional	2,899	977	1,409	2,410	5,843	19,176	32,714	19,355
sa gross roe	2,899	977	1,409	2,410	5,843	19,176	32,714	19,355
sa 1000 resamples, 0.5 of data	3,378	1,142	1,663	2,816	6,107	18,723	33,829	19,355
gross roe	-4,419	-2,195	-8,772	-4,408	-1,156	38,777	19,355	19,355
1000 resamples, 0.5 of data	-270	-200	-829	-319	1,038	19,936	19,355	19,355

Table 1: Gross traditional stand-alone vs natural allocation margin by line.

- Difference between traditional (=sa gross roe) and sa base reflects shape of risk by line compared to the total
 - Capital amount same: stand-alone VaR by line for both methods
 - Distortion charges lower volatility non-cat lines a **higher** ROE because they use relatively more expensive equity capital
- Last two rows show natural allocation margins, which are additive (sop = total)
- sa base compared to natural base reflects
 - Non-cat lines worse off with pooling
 - Compensated with negative margins
- sa gross roe compared to gross roe reflects impact of natural allocation

Net Traditional Stand-Alone vs Natural Allocation Margin by Line

method	BOP	CAuto	PAuto	Property	SCS	Wind	sop	total
traditional	3,372	1,136	1,639	2,804	3,885	4,289	17,126	6,553
sa net roe	3,372	1,136	1,639	2,804	3,885	4,289	17,126	6,553
sa 1000 resamples, 0.5 of data	3,378	1,142	1,663	2,816	3,911	4,277	17,188	6,553
net roe	-992	-1,262	-5,278	-1,705	4,802	10,966	6,553	6,553
1000 resamples, 0.5 of data	1,170	91	175	764	1,579	2,774	6,553	6,553

Table 2: Net traditional stand-alone vs natural allocation margin by line.

- traditional is very close to sa base
 - Tails of all lines comparable and base distortion close to constant ROE except in tail; all lines now thin tailed so pricing by layer has little impact
- sa base compared to base: greater balance in net book removes pooling penalty for non-cat lines seen in gross
 - All lines comparably risky
 - All lines pay positive margin
 - Better coverage makes all non-cat lines more expensive than in gross portfolio
- net roe distortion applied to net with natural allocation: tail-heavy focus still produces negative margins

Summary of Gross and Net Margin and Assets: Natural Allocation, Base

stat	kind	BOP	CAuto	PAuto	Property	SCS	Wind	total
Assets	Gross	20,378	9,743	38,774	20,039	12,543	154,373	255,850
	Net	28,193	10,847	42,157	25,521	15,254	21,027	143,000
	difference	-7,816	-1,104	-3,383	-5,482	-2,711	133,346	112,850
Leverage	Gross	29	-221	-113	51	0.925	0.186	0.863
	Net	3.01	13	20	4.36	0.707	0.351	2.66
	difference	26	-234	-133	47	0.218	-0.165	-1.8
Loss Ratio	Gross	1.01	1.02	1.02	1.02	0.828	0.177	0.837
	Net	0.945	0.991	0.996	0.963	0.75	0.492	0.937
	difference	0.069	0.0294	0.0256	0.053	0.0778	-0.315	-0.1
Margin	Gross	-270	-200	-829	-319	1,038	19,936	19,355
	Net	1,170	91	175	764	1,579	2,774	6,553
	difference	-1,440	-291	-1,004	-1,083	-541	17,162	12,802
ROE	Gross	-0.401	4.51	2.39	-0.832	0.159	0.153	0.141
	Net	0.166	0.12	0.0881	0.16	0.177	0.178	0.168
	difference	-0.567	4.39	2.31	-0.993	-0.0174	-0.025	-0.0269
Surplus	Gross	673	-44	-346	384	6,515	130,139	137,320
	Net	7,029	758	1,990	4,763	8,937	15,565	39,042
	difference	-6,356	-803	-2,336	-4,379	-2,422	114,574	98,278

Table 3: Summary of gross and net margin and assets: natural allocation, base distortion.

Summary of Gross and Net Margin and Assets: Natural Allocation, Constant Roe

stat	kind	BOP	CAuto	PAuto	Property	SCS	Wind	total
Assets	Gross	-15,775	-7,769	-31,022	-15,685	-4,362	318,098	255,850
	Net	13,101	1,222	3,293	8,138	38,135	78,959	143,000
	difference	-28,876	-8,991	-34,315	-23,823	-42,498	239,139	112,850
Leverage	Gross	-0.496	-0.501	-0.501	-0.498	-0.468	0.157	0.863
	Net	-3.22	-1.16	-1.1	-1.8	0.334	0.209	2.66
	difference	2.72	0.662	0.604	1.3	-0.801	-0.0525	-1.8
Loss Ratio	Gross	1.28	1.28	1.28	1.28	1.3	0.0998	0.837
	Net	1.05	1.14	1.15	1.09	0.497	0.197	0.937
	difference	0.232	0.137	0.129	0.19	0.805	-0.0971	-0.1
Margin	Gross	-4,419	-2,195	-8,772	-4,408	-1,156	38,777	19,355
	Net	-992	-1,262	-5,278	-1,705	4,802	10,966	6,553
	difference	-3,427	-933	-3,494	-2,702	-5,958	27,810	12,802
ROE	Gross	0.141	0.141	0.141	0.141	0.141	0.141	0.141
	Net	0.168	0.168	0.168	0.168	0.168	0.168	0.168
	difference	-0.027	-0.0269	-0.0269	-0.027	-0.0269	-0.0269	-0.0269
Surplus	Gross	-31,331	-15,561	-62,199	-31,252	-8,196	275,023	137,320
	Net	-5,902	-7,514	-31,421	-10,152	28,596	65,304	39,042
	difference	-25,429	-8,047	-30,778	-21,100	-36,792	209,718	98,278

Table 4: Summary of gross and net margin and assets: natural allocation, constant ROE distortion. Gross and net calibrated to different ROEs, determined by applying base to gross and net.

Offsetting Dynamics

- At the portfolio level, from gross to net
 - Asset levels decrease materially
 - Margins decrease materially
 - ROE increases: net book more concentrated in higher-return (lower) asset layers
 - Indeterminate effect in general
- By line, dynamics driven by shifting **relative** volatility
 - Only SCS and Wind covered by reinsurance programs
 - ineffective diversification in gross portfolio skews margin to cat lines; non-cat lines, penalized by pooling, receive negative margin in compensation
 - Effective diversification in net portfolio: margin allocated more uniformly across lines
 - Non-cat lines benefit from purchase of cat reinsurance, receive better protection in the net book, and pay a higher premium as a result, even though they are not covered directly by the reinsurance!

Gross and Net Loss Ratios by Method by Line

view	method	BOP	CAuto	PAuto	Property	SCS	Wind	sop	total
Gross	traditional	0.8733	0.911	0.966	0.8924	0.4561	0.1788	0.7517	0.8367
	sa 1000 resamples, 0.5 of data	0.8554	0.8975	0.9601	0.8765	0.4452	0.1823	0.7454	0.8367
	gross roe	1.284	1.282	1.281	1.283	1.301	0.09979	0.8367	0.8367
	1000 resamples, 0.5 of data	1.014	1.02	1.021	1.016	0.8278	0.1774	0.8367	0.8367
Net	traditional	0.8556	0.8979	0.9606	0.877	0.5485	0.3827	0.8504	0.937
	sa 1000 resamples, 0.5 of data	0.8554	0.8975	0.9601	0.8765	0.5469	0.3834	0.8499	0.937
	net roe	1.052	1.144	1.152	1.093	0.4966	0.1969	0.937	0.937
	1000 resamples, 0.5 of data	0.9447	0.991	0.9956	0.9632	0.75	0.4921	0.937	0.937

Table 5: Gross and net loss ratios by method by line.

- Exhibit summarizes loss ratios implied by margins (shown previously)
- `traditional` and `sa` base methods nearly identical on net basis; distortion is close to constant ROE for thin tailed lines
- `sa` vs. natural allocation base difference for net reflects relative consumption of capital
- More material difference for gross, driven by heterogeneous volatility
- `traditional` vs. `gross/net roe` shows difference between natural allocation and stand-alone

Gross to Net Transition: Summary

In total, gross to net shows...

- Less capital being used more efficiently = nearer peak utilization
- Peak utilization = higher unit cost (higher ROE): engine running faster
- (Higher unit cost) \times (fewer units) = indeterminate impact

By line, gross to net shows...

- Thin-tailed lines hurt by pooling with thick (ineffective diversification): negative margins compensate
- All lines comparably risky in net portfolio leading to **effective diversification** and positive margins paid by thin tailed lines
- Reinsurance alters economic value received by non-reinsured lines!

Gross Loss Ratio by Cat Bond Distortion

distortion	PAuto	CAuto	Property	BOP	total	SCS	Wind
base	1.021	1.02	1.016	1.014	0.837	0.828	0.177
average	1.003	1.003	1.001	0.999	0.952	0.907	0.483
regression	1.029	1.028	1.023	1.019	0.93	0.956	0.311
2009	1.016	1.015	1.011	1.008	0.849	0.798	0.198
2010	1.034	1.033	1.03	1.028	0.889	0.874	0.23
2011	1.011	1.01	1.007	1.005	0.891	0.852	0.263
2012	1.051	1.05	1.046	1.043	0.831	0.829	0.156
2013	1.013	1.012	1.007	1.004	0.907	0.881	0.294
2014	1.01	1.01	1.008	1.007	0.932	0.944	0.358
2015	1.005	1.004	1.002	1.001	0.946	0.911	0.439
2016	1.004	1.004	1	0.997	0.94	0.897	0.419
2017	1.004	1.003	1	0.998	0.945	0.908	0.443
2018	1.009	1.008	1.004	1.002	0.922	0.927	0.334
2019	1.002	1.001	0.994	0.989	0.931	0.835	0.413
2020	1.022	1.021	1.017	1.015	0.857	0.846	0.199
gross roe	1.281	1.282	1.283	1.284	0.837	1.301	0.0998

Table 6: Gross loss ratio by line using the natural allocation based on distortions parameterized to cat bond prices from different period, plus the base and constant ROE.

Gross Loss Ratio by Cat Bond Distortion

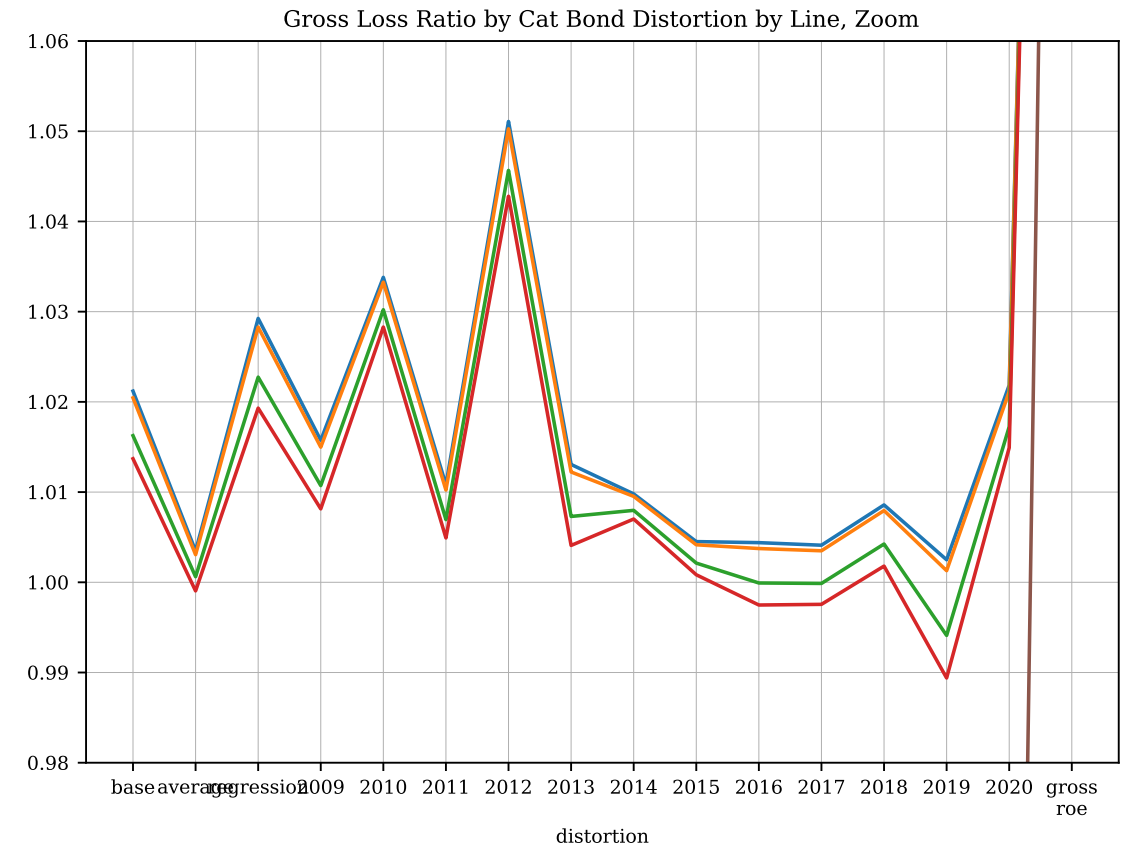
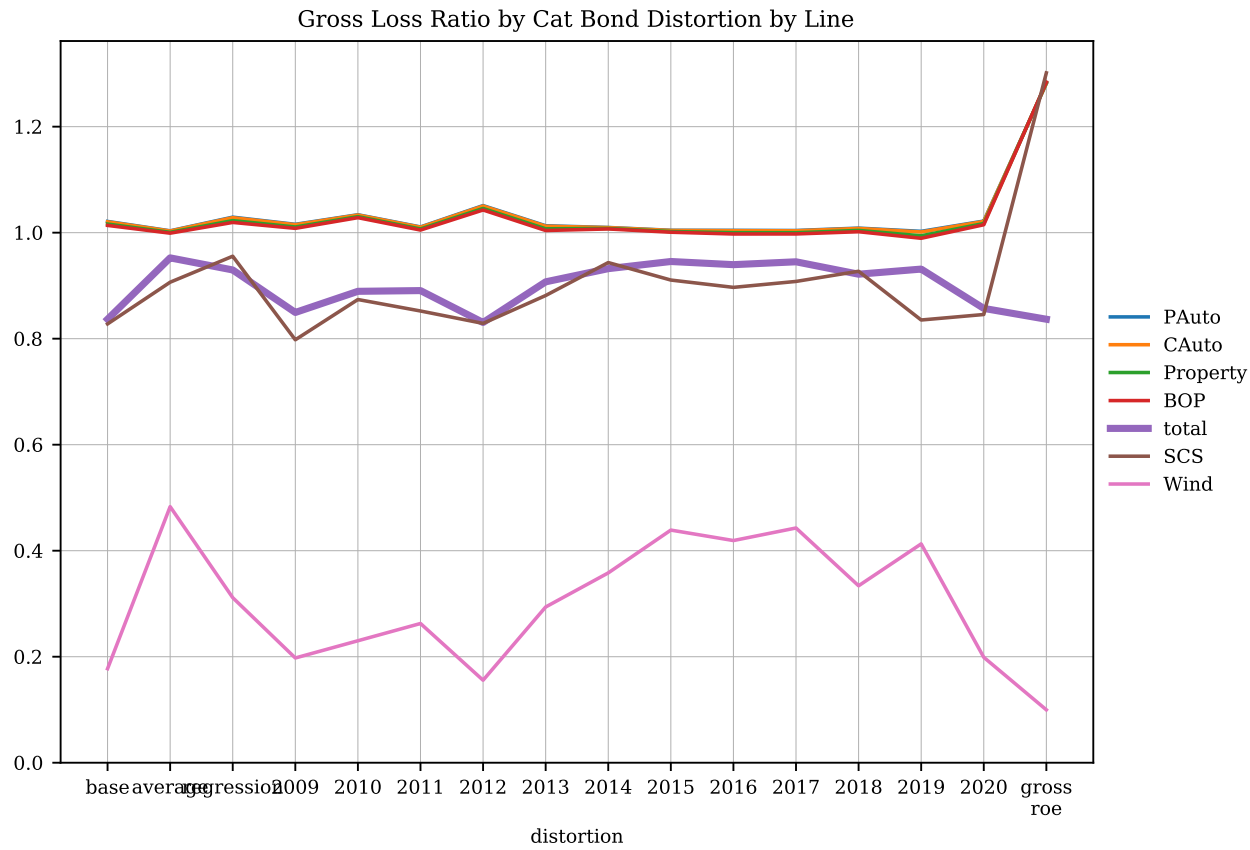


Figure 1: Right hand plot shows non-cat lines, pricing is consistent ordering of all non-cat lines. Lines sorted by loss ratio using base distortion. The cat market cycle is evident.

Net Loss Ratio by Cat Bond Distortion

distortion	PAuto	CAuto	Property	BOP	total	SCS	Wind
base	0.996	0.991	0.963	0.945	0.937	0.75	0.492
average	0.998	0.997	0.988	0.982	0.98	0.913	0.777
regression	1.003	1.001	0.987	0.977	0.972	0.857	0.65
2009	0.995	0.991	0.965	0.948	0.941	0.766	0.515
2010	1.001	0.998	0.978	0.965	0.957	0.805	0.563
2011	0.997	0.994	0.975	0.963	0.958	0.824	0.604
2012	1.002	0.997	0.967	0.946	0.933	0.721	0.445
2013	0.998	0.995	0.978	0.967	0.963	0.841	0.634
2014	0.999	0.997	0.987	0.979	0.976	0.89	0.723
2015	0.998	0.997	0.988	0.981	0.979	0.907	0.762
2016	0.998	0.996	0.985	0.977	0.976	0.894	0.737
2017	0.998	0.996	0.986	0.979	0.978	0.901	0.753
2018	0.998	0.996	0.982	0.973	0.97	0.869	0.685
2019	0.996	0.994	0.978	0.968	0.967	0.864	0.685
2020	0.997	0.993	0.97	0.954	0.947	0.78	0.533
net roe	1.152	1.144	1.093	1.052	0.937	0.497	0.197

Table 7: Net loss ratio by line using the natural allocation based on distortions parameterized to cat bond prices from different period, plus the base and constant ROE. Tighter clustering of loss ratios evident. Fixed ROE distortion stands out as extreme outlier.

Net Loss Ratio by Cat Bond Distortion

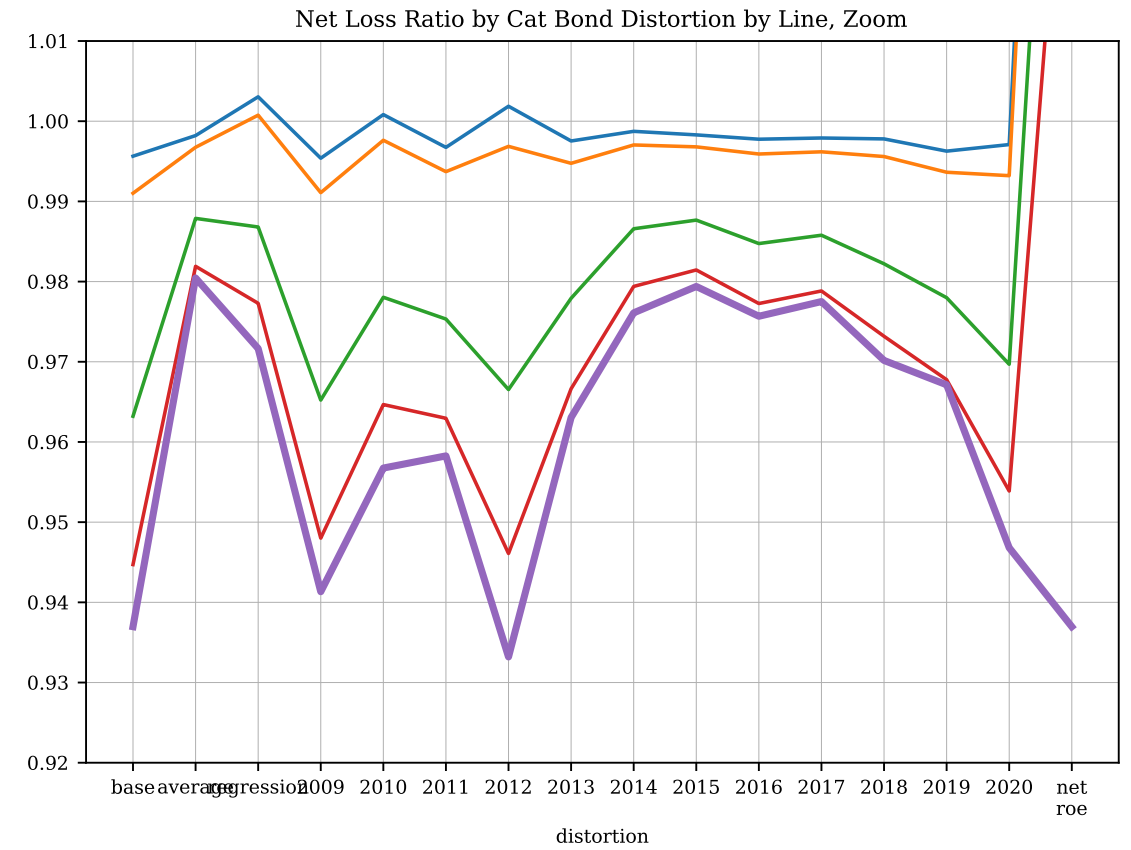
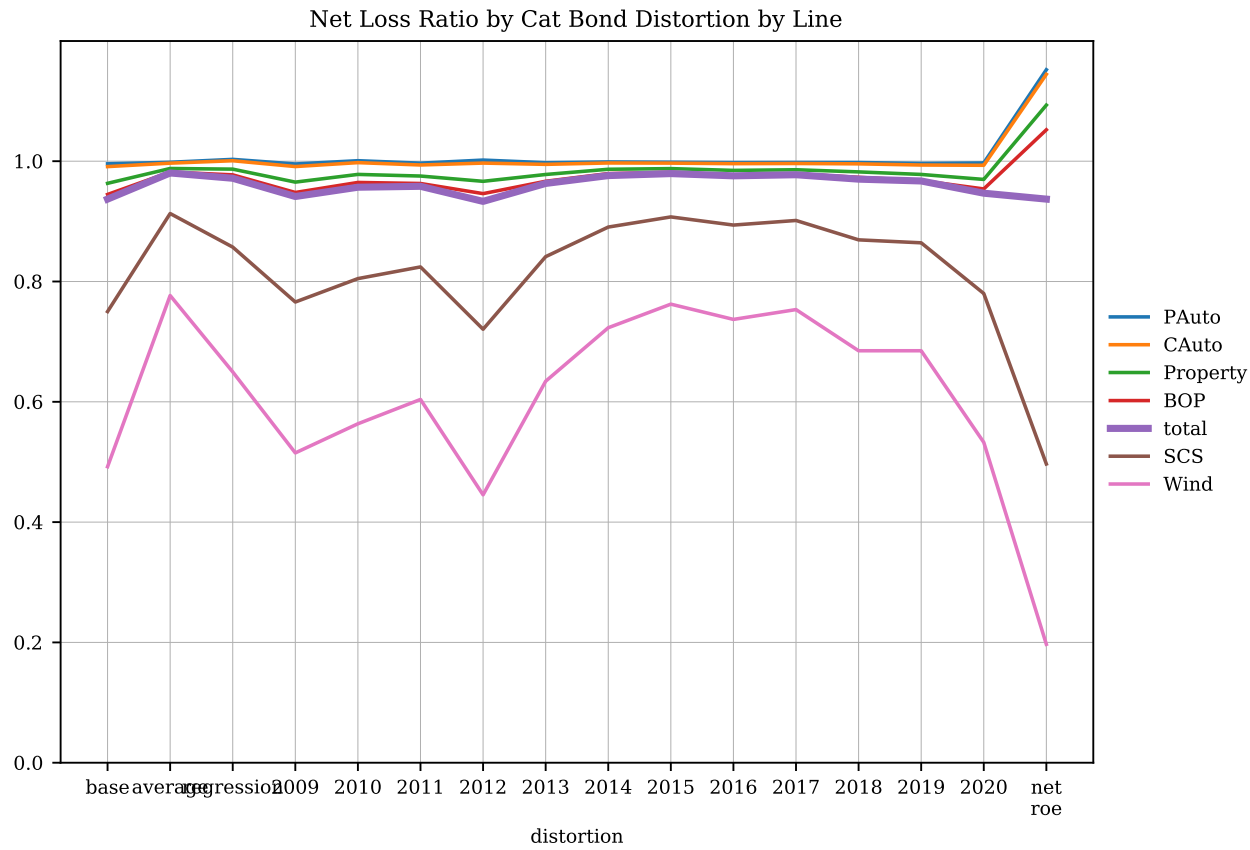


Figure 2: Right hand plot shows non-cat lines, pricing is consistent ordering of all non-cat lines. Lines sorted by loss ratio using base distortion. The cat market cycle is evident.

G.03. Line and Portfolio Multiline Pricing Across Multiple Methods

Road Map: Multiline Pricing

Analysis proceeds in three steps

In previous section:

1. Stand-alone traditional and distortion methods vs. natural allocation for base and constant roe distortions
 - Gross by line
 - Net by line
 - Explain gross to net transition for the natural allocation, base convex distortion
 - This section
2. Natural allocation by line for other cat bond calibrated distortions

In this section:

3. Distortion vs. multiline traditional methods by line
 - All methods calibrated to consistent total return

Overview

Recall objective: understand different approaches to pricing risk

- Final sections pull together all tools to address objective
- Compare variety of distortion and traditional methods, on gross and net bases
- Stand-alone pricing no longer relevant

Common calibration

- All pricing methods calibrated to gross ROE of (0.141) determined by base base distortion

Pricing Methods

Distortion methods

- base constant ROE, and parametric distortions
- Standard distortions including proportional hazard, Wang, and dual
- Natural allocation used to determine capital and margin by line
- Distortion methods prefixed Dist

What are traditional methods?

- Traditional methods characterized by a **constant ROE on capital** assumption, a RORAC approach
- Traditional methods defined are by an allocation procedure, e.g., scaled-VaR, coTVaR
- All traditional methods parameterized to a target ROE matching the base distortion method, separately for gross and net
- Include non-additive traditional methods such as Merton Perold for reference
- Traditional methods prefixed Trad (Trad* for non-additive methods)

Traditional methods

- VaR, TVaR, EDP: apply stand-alone and add-up capital (non-additive)
- Equal Risk (EqRisk) VaR solves for p^* so $\sum_i \text{VaR}_{p^*}(X_i) = \text{VaR}_{0.996}(X)$, and similarly for TVaR and EPD
- Scaled VaR = peanut-butter spread: re-scale stand-alone VaR to portfolio total
- MerPer = Merton-Perold, marginal by unit using VaR capital
- coTVaR = $E[X_i | X > \text{VaR}]$ (assumes X continuous)
- covar = covariance allocation, margin proportional to variance
- Prefixed Trad or Trad* for non-additive methods

Calibrated Standard Distortions

method	S	ROE	param	error
Proportional Hazard	0.004	0.141	0.52	2.62e-10
Wang-normal	0.004	0.141	0.832	8.44e-07
Capped Loglinear	0.004	0.141	0.537	1.46e-11
Leverage Equivalent Pricing	0.004	0.141	0.986	-1.8e-09
Linear Yield	0.004	0.141	3.06	-1.89e-08
Capped Linear	0.004	0.141	2.36	-2.91e-11
Dual Moment	0.004	0.141	4.08	-1.93e-06
Constant ROE	0.004	0.141	0.141	0
Tail VaR	0.004	0.141	0.638	0
Weighted TVaR	0.004	0.141	0.124	1.46e-11

Table 8: Shape parameter for standard distortions calibrated to 0.141 gross ROE at 0.996 VaR capital, as as produced by the base 'convex' ('base') distortion. Two parameter distortions assume 3 percent minimum ROL (c.f., Florida hurricane minimums). Weighted TVaR uses $p = 0$ (mean), $p = 0.996$. Achieved $S(x)$ is very close to expected $0.004 = 1 - 0.996$, corresponding to 250 year return period assets. Error column shows absolute approximation error to desired price.

Calibrated Standard Distortions

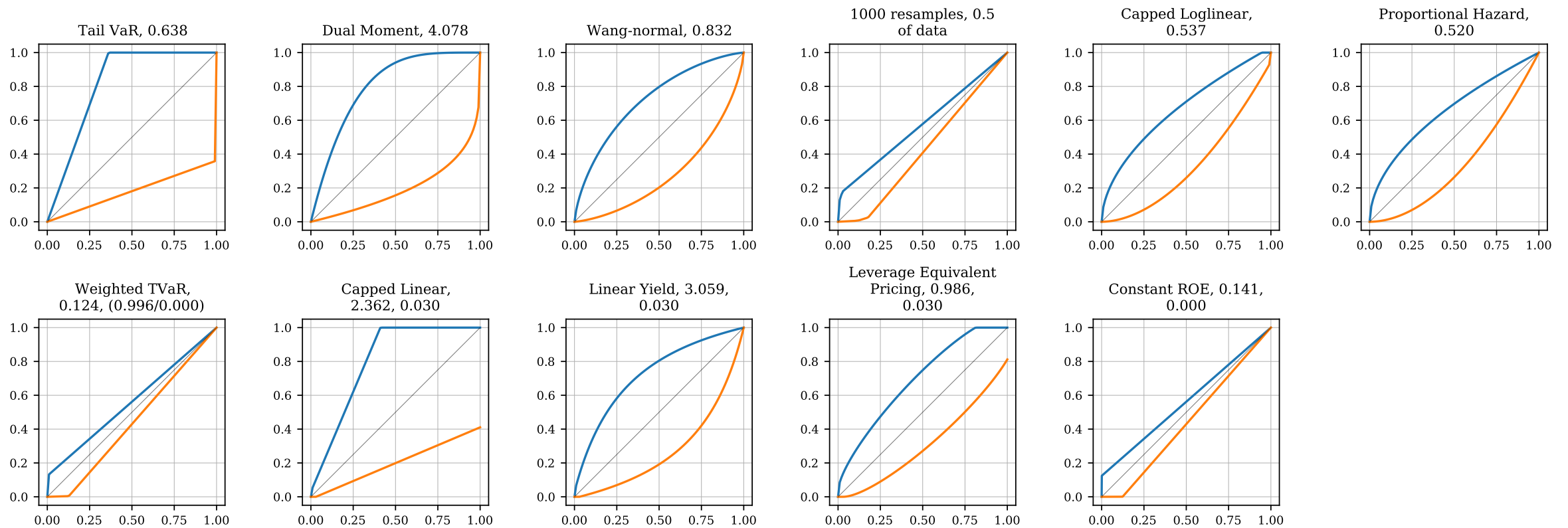


Figure 3: Standard distortions calibrated to produce same gross ROE 14.1% as base (upper right) on total portfolio. Distortions are sorted by implied cost of 1000 year event, given by $g(0.001)$. TVaR distortion matches premium, not measuring tail risk; note low $p = 0.638$. Material differences in mid-body behavior, $s > 0.1$, results in different prices for cat and non-cat risk. Constant ROE and base, and to a lesser extent Leverage Equivalent Pricing (LEP), produce cheapest non-cat pricing and most expensive cat pricing.

Calibrated Standard Distortions, Log Scale

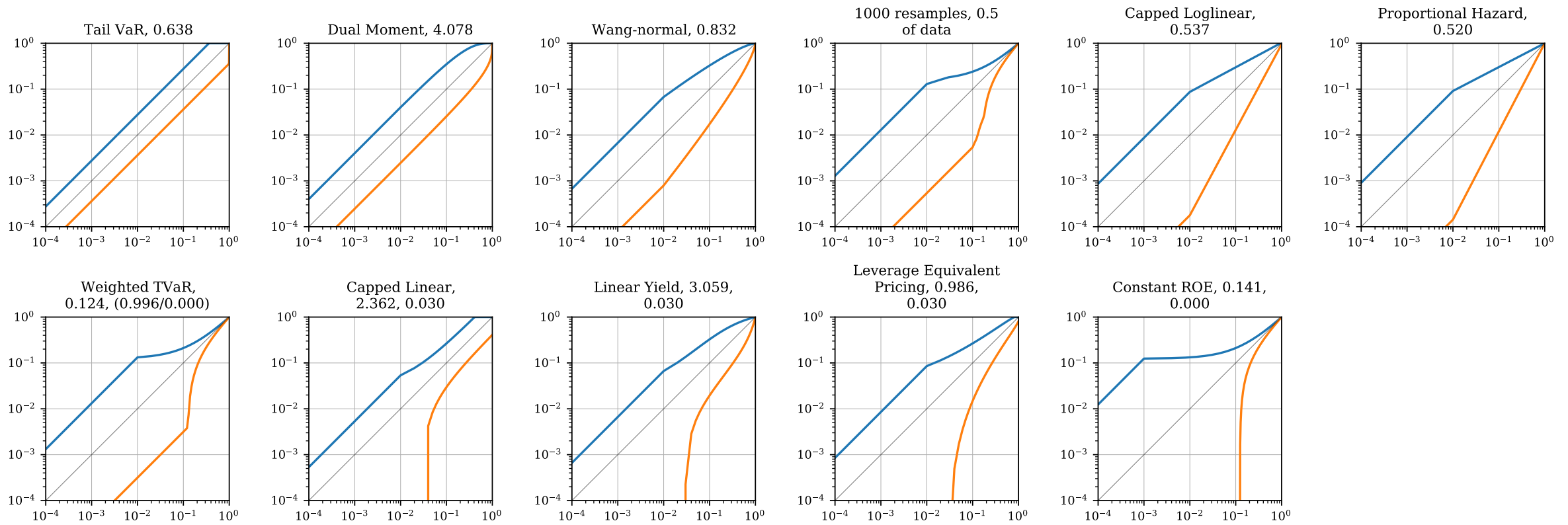


Figure 4: Calibrated standard distortion on a log scale, emphasizing behavior for small s . Log scale emphasizes behavior for small s , which determines cost of tail risk. The ordering is more obvious. Shows the same distortions as the previous plot.

Roe vs Exceeding Probability for Standard Distortions

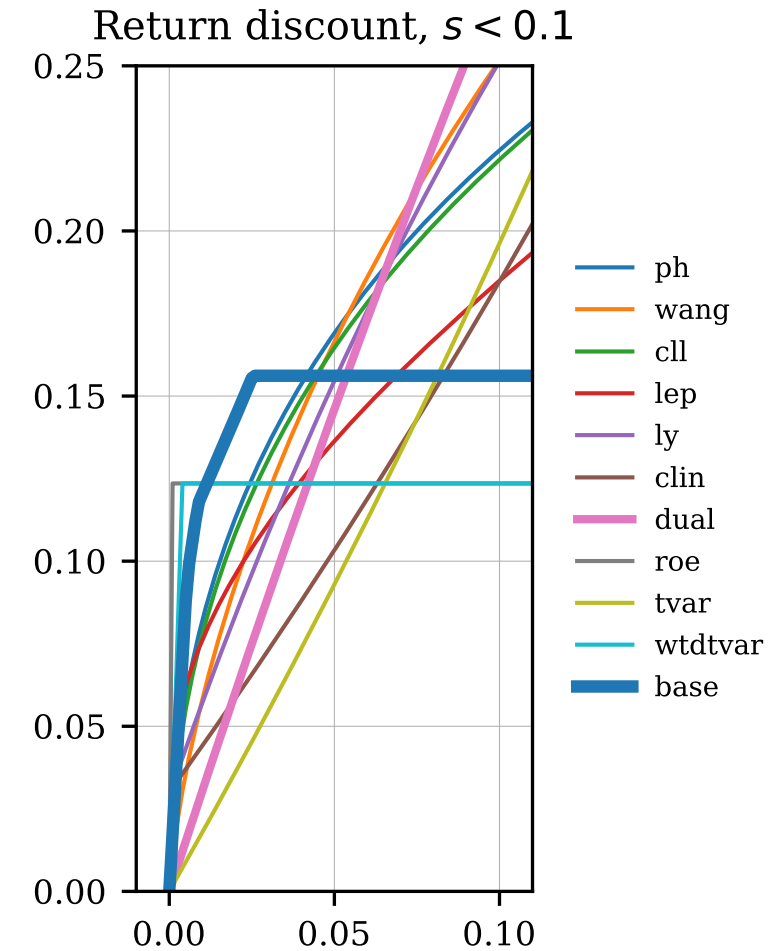
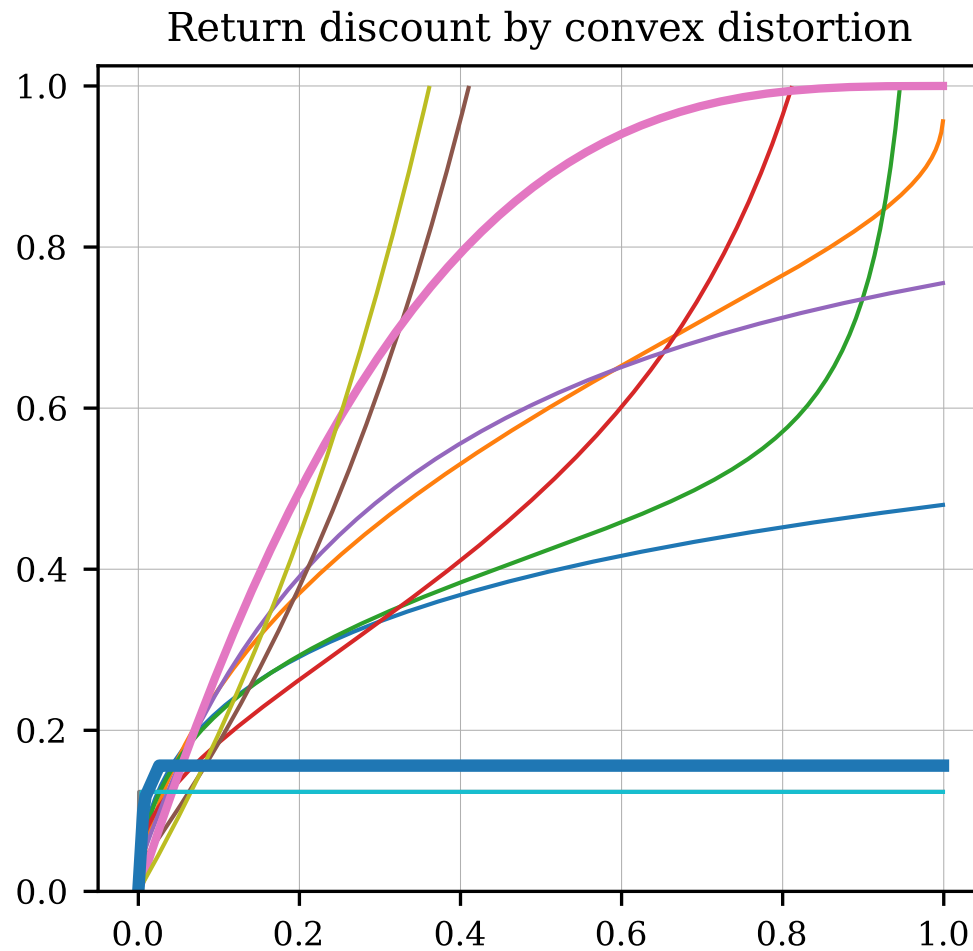


Figure 5: ROE discount vs. exceeding probability. Standard parametric distortions calibrated to produce 0.141 total ROE. Unlike convex envelope distortions calibrated to cat bond prices, standard distortions exhibit increasing or unbounded maximum ROE for larger s

ROE vs. Exceeding Probability

Distinguishing distortion behavior

- Constant ROE distortion produces high ROEs for small s , i.e., large losses
- Constant ROE and base have capped ROE for all s , matching an intuitive credit yield curve
- The dual and (pricing, low p) TVaR distortions have the lowest ROEs for large losses
- All other standard parametric distortions produce very high ROEs for larger s , i.e., smaller losses

Gross Loss Ratio, by Method by Line

kind	line	BOP	CAuto	PAuto	Property	SCS	Wind	total
Dist	1000 resamples, 0.5 of data	1.01	1.02	1.02	1.02	0.828	0.177	0.837
	clin	0.902	1.02	1.04	0.941	0.723	0.22	0.837
	cII	0.945	1	1.01	0.965	0.708	0.213	0.837
	dual	0.848	0.962	0.977	0.887	0.603	0.334	0.837
	lep	0.982	1.05	1.06	1.01	0.797	0.175	0.837
	ly	0.95	1.04	1.05	0.981	0.709	0.195	0.837
	ph	0.957	1.01	1.01	0.976	0.715	0.205	0.837
	roe	1.28	1.28	1.28	1.28	1.3	0.0998	0.837
	tvar	0.835	0.956	0.972	0.875	0.638	0.424	0.853
	wang	0.903	0.983	0.993	0.93	0.648	0.255	0.837
	wtdtvar	1.04	1.04	1.04	1.04	1.02	0.158	0.837
	Trad	EqRiskEPD	1.01	1.01	1.02	1.01	0.835	0.18
EqRiskTVaR		0.899	0.929	0.972	0.914	0.602	0.3	0.837
EqRiskVaR		0.892	0.924	0.97	0.908	0.582	0.317	0.837
ScaledEPD		1.1	1.11	1.11	1.1	0.863	0.136	0.837
ScaledTVaR		0.938	0.967	1.01	0.953	0.586	0.241	0.837
ScaledVaR		0.939	0.969	1.01	0.954	0.56	0.244	0.837
coTVaR		0.992	0.998	0.999	0.994	0.761	0.198	0.837
covar		0.963	0.99	0.994	0.972	0.854	0.204	0.837
Trad*	EPD	0.921	0.95	0.987	0.936	0.376	0.024	0.346
	MerPer	0.875	0.781	0.934	0.876	0.631	0.241	0.781
	TVaR	0.882	0.917	0.968	0.899	0.499	0.187	0.763
	VaR	0.873	0.911	0.966	0.892	0.461	0.183	0.752

Table 9: Gross loss ratio, by method by line. Trad* methods are not additive and have a lower overall loss ratio

Net Loss Ratio, by Method by Line

kind	line	BOP	CAuto	PAuto	Property	SCS	Wind	total
Dist	1000 resamples, 0.5 of data	0.945	0.991	0.996	0.963	0.75	0.492	0.937
	clin	0.86	0.986	1	0.903	0.603	0.349	0.878
	cll	0.897	0.977	0.987	0.925	0.696	0.454	0.907
	dual	0.837	0.956	0.972	0.877	0.646	0.435	0.871
	lep	0.908	1	1.01	0.941	0.633	0.355	0.904
	ly	0.892	0.998	1.01	0.93	0.61	0.34	0.892
	ph	0.904	0.98	0.989	0.932	0.697	0.45	0.911
	roe	1.05	1.13	1.13	1.08	0.53	0.219	0.945
	tvar	0.833	0.954	0.97	0.873	0.666	0.477	0.875
	wang	0.872	0.968	0.98	0.905	0.676	0.447	0.892
	wtdtvar	0.956	0.996	1	0.972	0.766	0.505	0.945
	Trad	EqRiskEPD	0.945	0.973	1	0.96	0.611	0.377
EqRiskTVaR		0.902	0.93	0.971	0.916	0.733	0.587	0.911
EqRiskVaR		0.9	0.928	0.971	0.914	0.748	0.592	0.911
ScaledEPD		0.943	0.998	1.07	0.971	0.514	0.316	0.911
ScaledTVaR		0.918	0.965	1.03	0.941	0.579	0.401	0.911
ScaledVaR		0.917	0.963	1.03	0.94	0.582	0.406	0.911
coTVaR		0.921	0.983	0.989	0.946	0.682	0.41	0.911
covar		0.884	0.968	0.979	0.912	0.743	0.554	0.911
Trad*	EPD	0.781	0.851	0.949	0.816	0.346	0.196	0.741
	MerPer	0.945	0.957	0.986	0.957	0.8	0.592	0.941
	TVaR	0.803	0.858	0.944	0.83	0.452	0.295	0.794
	VaR	0.803	0.858	0.944	0.83	0.456	0.301	0.796

Table 10: Net loss ratio, by method by line. Net loss ratios are higher than gross. All Dist methods have different LRs, they are calibrated to the same total gross premium. All Trad methods have the same LRs because they are calibrated to the same net ROE. Loss ratios change for lines with no reinsurance; the relative risk of lines on a net basis is different than gross.

Gross Margin, by Method by Line

kind	line	BOP	CAuto	PAuto	Property	SCS	Wind	total
Dist	1000 resamples, 0.5 of data	-270	-200	-829	-319	1,038	19,936	19,355
	clin	2,178	-210	-1,464	1,248	1,916	15,248	19,355
	cll	1,164	-30	-410	718	2,054	15,860	19,355
	dual	3,579	398	954	2,556	3,288	8,580	19,355
	lep	376	-505	-2,376	-107	1,269	20,246	19,355
	ly	1,058	-373	-1,903	390	2,044	17,698	19,355
	ph	902	-87	-589	500	1,992	16,638	19,355
	roe	-4,419	-2,195	-8,772	-4,408	-1,156	38,777	19,355
	tvar	3,954	461	1,137	2,853	2,832	5,845	17,082
	wang	2,157	173	270	1,493	2,705	12,556	19,355
	wtdtvar	-726	-380	-1,529	-739	-99	22,829	19,355
Trad	EqRiskEPD	-119	-137	-786	-204	984	19,618	19,355
	EqRiskTVaR	2,237	763	1,135	1,875	3,293	10,053	19,355
	EqRiskVaR	2,425	823	1,217	2,029	3,584	9,259	19,355
	ScaledEPD	-1,804	-956	-4,069	-1,860	792	27,252	19,355
	ScaledTVaR	1,314	337	-340	986	3,520	13,539	19,355
	ScaledVaR	1,307	321	-470	964	3,918	13,316	19,355
	coTVaR	156	20	52	112	1,565	17,452	19,356
	covar	775	99	253	570	854	16,804	19,355
Trad*	EPD	1,723	522	529	1,368	8,269	174,799	187,211
	MerPer	2,851	2,800	2,817	2,838	2,917	13,534	27,756
	TVaR	2,684	906	1,324	2,238	5,018	18,636	30,805
	VaR	2,900	978	1,415	2,412	5,832	19,161	32,698

Table 11: Gross margin, by method by line. Wind is the only line with an unequivocally positive margin. PAuto has lowest the volatility and margins. The constant ROE Dist roe distortion produces most extreme margins: it has a very high minimum ROL and is extremely sensitive to tail risk.

Net Margin, by Method by Line

kind	line	BOP	CAuto	PAuto	Property	SCS	Wind	total
Dist	1000 resamples, 0.5 of data	1,170	91	175	764	1,579	2,774	6,553
	clin	3,263	138	-132	2,155	3,122	5,014	13,565
	cll	2,302	238	542	1,610	2,069	3,231	9,992
	dual	3,893	461	1,161	2,800	2,601	3,497	14,413
	lep	2,037	-26	-577	1,247	2,743	4,886	10,316
	ly	2,422	19	-438	1,503	3,025	5,229	11,765
	ph	2,114	200	438	1,451	2,056	3,285	9,544
	roe	-891	-1,115	-4,659	-1,515	4,197	9,596	5,633
	tvar	4,020	485	1,225	2,914	2,376	2,948	13,968
	wang	2,937	333	816	2,089	2,275	3,326	11,775
	wtdtvar	920	43	17	572	1,449	2,632	5,633
Trad	EqRiskEPD	1,153	273	-176	833	3,018	4,444	9,544
	EqRiskTVaR	2,160	758	1,176	1,836	1,727	1,888	9,544
	EqRiskVaR	2,210	775	1,205	1,875	1,593	1,855	9,544
	ScaledEPD	1,201	24	-2,575	598	4,479	5,817	9,544
	ScaledTVaR	1,785	364	-1,308	1,243	3,438	4,023	9,544
	ScaledVaR	1,821	381	-1,266	1,275	3,397	3,937	9,544
	coTVaR	1,715	172	430	1,151	2,214	3,864	9,547
	covar	2,624	335	857	1,928	1,638	2,162	9,544
Trad*	EPD	5,598	1,752	2,157	4,502	8,946	11,025	33,981
	MerPer	1,163	450	577	901	1,185	1,855	6,132
	TVaR	4,906	1,648	2,385	4,081	5,755	6,422	25,197
	VaR	4,910	1,654	2,388	4,083	5,654	6,251	24,940

Table 12: Net margin, by method by line. The total margin is lower for net: less capital, higher per unit cost (ROE). Fewer margins are negative reflecting more effective diversification in net portfolio. There are 35 negative gross margins and 12 negative net margins.

Gross Allocated Capital, by Method by Line

kind	line	BOP	CAuto	PAuto	Property	SCS	Wind	total
Dist	1000 resamples, 0.5 of data	673	-44	-346	384	6,515	130,139	137,320
	clin	4,517	2,037	8,055	4,343	2,563	111,925	137,320
	cII	10,280	4,783	18,983	10,002	6,233	87,040	137,320
	dual	12,962	6,027	23,916	12,609	8,398	73,408	137,320
	lep	5,828	2,697	10,697	5,659	3,337	106,480	137,320
	ly	6,579	3,201	12,783	6,496	3,693	101,467	137,320
	ph	9,071	3,718	14,509	8,452	7,008	94,562	137,320
	roe	-31,331	-15,561	-62,199	-31,252	-8,196	275,023	137,320
	tvar	13,333	6,190	24,558	12,962	8,624	73,927	139,594
	wang	11,781	5,462	21,667	11,449	7,351	79,609	137,320
	wtdtvar	-5,149	-2,695	-10,837	-5,236	-703	161,939	137,320
Trad	EqRiskEPD	-843	-975	-5,575	-1,444	6,978	139,179	137,320
	EqRiskTVaR	15,867	5,414	8,051	13,299	23,364	71,324	137,320
	EqRiskVaR	17,201	5,840	8,633	14,396	25,426	65,692	137,320
	ScaledEPD	-12,795	-6,780	-28,871	-13,195	5,616	193,345	137,320
	ScaledTVaR	9,321	2,391	-2,414	6,998	24,970	96,054	137,320
	ScaledVaR	9,272	2,278	-3,336	6,837	27,798	94,472	137,320
	coTVaR	1,107	139	368	795	11,101	123,813	137,321
	covar	5,502	702	1,797	4,042	6,060	119,220	137,320
Trad*	EPD	12,226	3,703	3,756	9,705	58,663	1,240,137	1,328,189
	MerPer	20,225	19,863	19,984	20,137	20,693	96,018	196,919
	TVaR	19,039	6,429	9,395	15,875	35,600	132,213	218,551
	VaR	20,575	6,935	10,036	17,113	41,378	135,940	231,978

Table 13: Gross allocated capital, by method by line. The natural allocation for distortion methods, assumes equal ROE by line within layer, the only choice consistent with law invariance A negative margin begets a negative capital allocation. The allocation varies by method for remaining approaches. Trad* methods are not additive

Net Allocated Capital, by Method by Line

kind	line	BOP	CAuto	PAuto	Property	SCS	Wind	total
Dist	1000 resamples, 0.5 of data	7,029	758	1,990	4,763	8,937	15,565	39,042
	clin	5,215	1,894	7,332	4,587	4,728	8,261	32,030
	cll	7,069	2,727	10,597	6,409	3,731	5,070	35,603
	dual	6,329	2,508	9,773	5,800	2,988	3,784	31,182
	lep	6,073	2,242	8,691	5,387	4,824	8,052	35,279
	ly	5,803	2,407	9,480	5,348	4,100	6,682	33,830
	ph	7,349	2,165	8,043	6,152	4,977	7,365	36,051
	roe	-6,317	-7,903	-33,031	-10,739	29,765	68,050	39,962
	tvar	6,427	2,551	9,942	5,893	3,014	3,800	31,627
	wang	6,826	2,656	10,330	6,215	3,381	4,411	33,819
	wtdtvar	6,530	304	124	4,058	10,280	18,665	39,962
	Trad	EqRiskEPD	4,353	1,031	-666	3,145	11,401	16,786
EqRiskTVaR		8,160	2,863	4,440	6,935	6,523	7,130	36,051
EqRiskVaR		8,346	2,927	4,553	7,081	6,019	7,007	36,051
ScaledEPD		4,536	91	-9,726	2,258	16,919	21,972	36,051
ScaledTVaR		6,741	1,374	-4,942	4,694	12,988	15,197	36,051
ScaledVaR		6,877	1,441	-4,783	4,814	12,831	14,870	36,051
coTVaR		6,479	650	1,622	4,349	8,363	14,596	36,060
covar		9,913	1,264	3,237	7,283	6,187	8,166	36,051
Trad*	EPD	21,146	6,619	8,149	17,007	33,790	41,644	128,354
	MerPer	4,393	1,702	2,181	3,404	4,477	7,007	23,163
	TVaR	18,532	6,226	9,008	15,415	21,738	24,256	95,174
	VaR	18,546	6,248	9,020	15,422	21,358	23,611	94,205

Table 14: Net allocated capital, by method by line. Shown for reference.

Gross Leverage, by Method by Line

kind	line	BOP	CAuto	PAuto	Property	SCS	Wind	total
Dist	1000 resamples, 0.5 of data	29.3	-221	-113	51.2	0.925	0.186	0.863
	clin	4.9	4.8	4.78	4.89	2.69	0.175	0.863
	cll	2.06	2.08	2.08	2.07	1.13	0.232	0.863
	dual	1.82	1.72	1.71	1.79	0.986	0.175	0.863
	lep	3.49	3.52	3.51	3.51	1.88	0.231	0.863
	ly	3.2	3	2.98	3.14	1.9	0.217	0.863
	ph	2.3	2.66	2.71	2.42	0.996	0.221	0.863
	roe	-0.496	-0.501	-0.501	-0.498	-0.468	0.157	0.863
	tvar	1.79	1.69	1.67	1.76	0.907	0.137	0.833
	wang	1.88	1.86	1.86	1.88	1.05	0.212	0.863
	wtdtvar	-3.74	-3.57	-3.55	-3.67	-6.96	0.168	0.863
Trad	EqRiskEPD	-23.5	-10.1	-7.03	-13.7	0.856	0.172	0.863
	EqRiskTVaR	1.4	1.99	5.1	1.64	0.355	0.201	0.863
	EqRiskVaR	1.3	1.85	4.77	1.53	0.337	0.206	0.863
	ScaledEPD	-1.42	-1.33	-1.24	-1.37	1.03	0.163	0.863
	ScaledTVaR	2.28	4.32	-16.4	3	0.341	0.186	0.863
	ScaledVaR	2.3	4.52	-11.8	3.06	0.32	0.186	0.863
	coTVaR	18.2	72.2	109	25.3	0.59	0.176	0.863
	covar	3.77	14.4	22.4	5.08	0.964	0.177	0.863
Trad*	EPD	1.77	2.84	10.8	2.2	0.226	0.144	0.216
	MerPer	1.13	0.644	2.14	1.13	0.382	0.186	0.645
	TVaR	1.19	1.69	4.39	1.4	0.281	0.173	0.595
	VaR	1.11	1.58	4.12	1.31	0.262	0.173	0.568

Table 15: Gross leverage, by method by line. Leverage is the ratio of premium to allocated capital. The total portfolio allocation in line with US statutory premium to surplus ratio and appropriate for a volatile book. Individual line leverage impossible to interpret, another sign of ineffective diversification. Reserve risk missing from analysis; a one-year model collapses reserve risk into premium risk by assuming losses known with certainty at end of year.

Net Leverage, by Method by Line

kind	line	BOP	CAuto	PAuto	Property	SCS	Wind	total
Dist	1000 resamples, 0.5 of data	3.01	13.3	20.2	4.36	0.707	0.351	2.66
	clin	4.46	5.35	5.44	4.83	1.66	0.932	3.46
	cII	3.15	3.75	3.82	3.37	1.82	1.17	3.02
	dual	3.77	4.17	4.21	3.93	2.46	1.63	3.59
	lep	3.63	4.45	4.53	3.94	1.55	0.941	3.05
	ly	3.86	4.16	4.17	4.02	1.89	1.18	3.23
	ph	3.01	4.71	5.03	3.49	1.37	0.811	2.97
	roe	-3.02	-1.12	-1.07	-1.72	0.3	0.181	2.58
	tvar	3.74	4.11	4.15	3.89	2.36	1.48	3.52
	wang	3.36	3.89	3.95	3.55	2.07	1.36	3.23
	wtdtvar	3.2	33.1	322	5.07	0.602	0.285	2.58
	Trad	EqRiskEPD	4.86	9.96	-59.8	6.62	0.68	0.425
EqRiskTVaR		2.72	3.76	9.27	3.15	0.991	0.642	2.97
EqRiskVaR		2.66	3.68	9.05	3.09	1.05	0.648	2.97
ScaledEPD		4.67	110	-3.85	9.12	0.545	0.387	2.97
ScaledTVaR		3.23	7.54	-7.83	4.52	0.63	0.442	2.97
ScaledVaR		3.17	7.2	-8.1	4.42	0.634	0.446	2.97
coTVaR		3.35	15.6	24.9	4.86	0.831	0.449	2.97
covar		2.28	8.17	12.6	3.01	1.03	0.594	2.97
Trad*	EPD	1.21	1.78	5.17	1.44	0.405	0.329	1.02
	MerPer	4.82	6.14	18.6	6.14	1.32	0.648	4.47
	TVaR	1.34	1.87	4.7	1.56	0.483	0.376	1.29
	VaR	1.34	1.86	4.7	1.56	0.487	0.379	1.3

Table 16: Net premium to allocated capital leverage, by method by line. Net total leverage higher than gross and varies by line because margin varies by line (total assets are the same for all additive methods). Individual line leverage comprehensible and intuitively reasonable. Low volatility PAuto can receive very high leverage.

Gross ROE, by Method by Line

kind	line	BOP	CAuto	PAuto	Property	SCS	Wind	total
Dist	tvar	0.297	0.0745	0.0463	0.22	0.328	0.0791	0.122
	dual	0.276	0.0661	0.0399	0.203	0.392	0.117	0.141
	clin	0.482	-0.103	-0.182	0.287	0.748	0.136	0.141
	wtdtvar	0.141	0.141	0.141	0.141	0.141	0.141	0.141
	roe	0.141	0.141	0.141	0.141	0.141	0.141	0.141
	1000 resamples, 0.5 of data	-0.401	4.51	2.39	-0.832	0.159	0.153	0.141
	wang	0.183	0.0317	0.0125	0.13	0.368	0.158	0.141
	ly	0.161	-0.116	-0.149	0.0601	0.553	0.174	0.141
	ph	0.0995	-0.0234	-0.0406	0.0591	0.284	0.176	0.141
	cII	0.113	-0.00621	-0.0216	0.0718	0.33	0.182	0.141
	lep	0.0646	-0.187	-0.222	-0.0188	0.38	0.19	0.141

Table 17: ROE varies by line for all distortions except constant ROE. Sorted by ROE for Wind (total ROEs are all equal). Traditional approaches produce the same loss ratio by line and are not shown.

Net Roe, by Method by Line

kind	line	BOP	CAuto	PAuto	Property	SCS	Wind	total
Dist	wtdtvar	0.141	0.14	0.136	0.141	0.141	0.141	0.141
	roe	0.141	0.141	0.141	0.141	0.141	0.141	0.141
	1000 resamples, 0.5 of data	0.166	0.12	0.0881	0.16	0.177	0.178	0.168
	ph	0.288	0.0926	0.0544	0.236	0.413	0.446	0.265
	lep	0.335	-0.0114	-0.0664	0.232	0.569	0.607	0.292
	clin	0.626	0.0727	-0.0181	0.47	0.66	0.607	0.423
	cII	0.326	0.0871	0.0512	0.251	0.554	0.637	0.281
	wang	0.43	0.125	0.079	0.336	0.673	0.754	0.348
	tvar	0.625	0.19	0.123	0.494	0.789	0.776	0.442
	ly	0.417	0.00791	-0.0462	0.281	0.738	0.782	0.348
	dual	0.615	0.184	0.119	0.483	0.871	0.924	0.462

Table 18: Net Return on equity ratios by method. Sorted by total ROE. Traditional approaches produce the same loss ratio by line and are not shown.

ROE by Method by Line

Hard to interpret

- ROEs by line are hard to interpret for distortion based methods
 - Positive ROE can occur as negative margin over negative capital
 - Negative capital: thin tail lines providing extreme tail protection to thick tailed lines Negative margin implies negative capital allocation for low asset levels.
- It is better to focus on loss ratio or margin amounts.
- Total ROE varies by distortion, even though all distortions calibrated to same gross ROE
- More concave (bowed) distortions show highest net ROEs—they are very sensitive to smaller losses (and may or may not be sensitive to large losses)
- Net book exhibits effective diversification and shows moderate and predominately positive ROEs by line across all methods

Gross and Net Pricing, by Method by Line

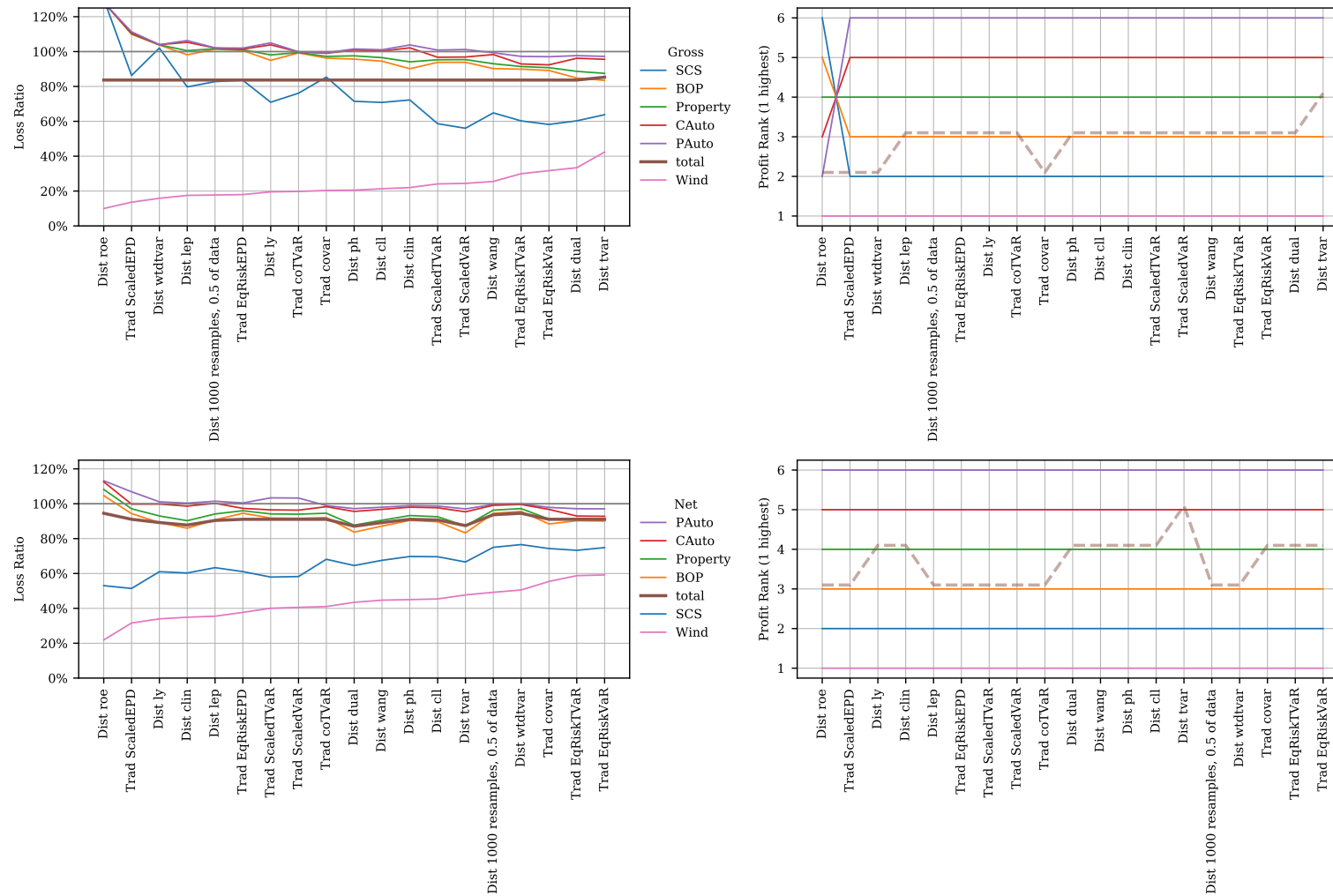


Figure 6: Pricing by distortion, additive allocations only. Top gross, bottom net; left targets, right ranks. Distortions ordered by pricing on PAuto. Lines ordered by pricing on first distortion. Distortion ordering differs gross to net. Total rank offset slightly, for clarity.

Gross and Net Pricing, by Method by Line

index	PAuto	CAuto	Property	BOP	total	SCS	Wind
EL	40,000	10,000	20,000	20,000	100,000	5,000	5,000
Avg LR	1.02	0.992	0.97	0.954	0.807	0.709	0.214
CV	0.1	0.25	0.3	0.35	0.35	1.47	6.52
P99.6	1.28	1.79	1.97	2.17	2.56	10.44	31.88

Table 19: Average loss ratio across all methods, by line, compared with CV of loss and ratio of 0.996 VaR to expected loss. Lines sorted by descending loss ratio, from cheapest to most expensive.

- Pricing and risk measures consistent
- Individual ranks on previous slide also consistent
 - Only order switch is constant ROE vs. all other methods for gross losses
- Gross and net orderings are consistent

Loss Ratio Ranges, by Method by Line

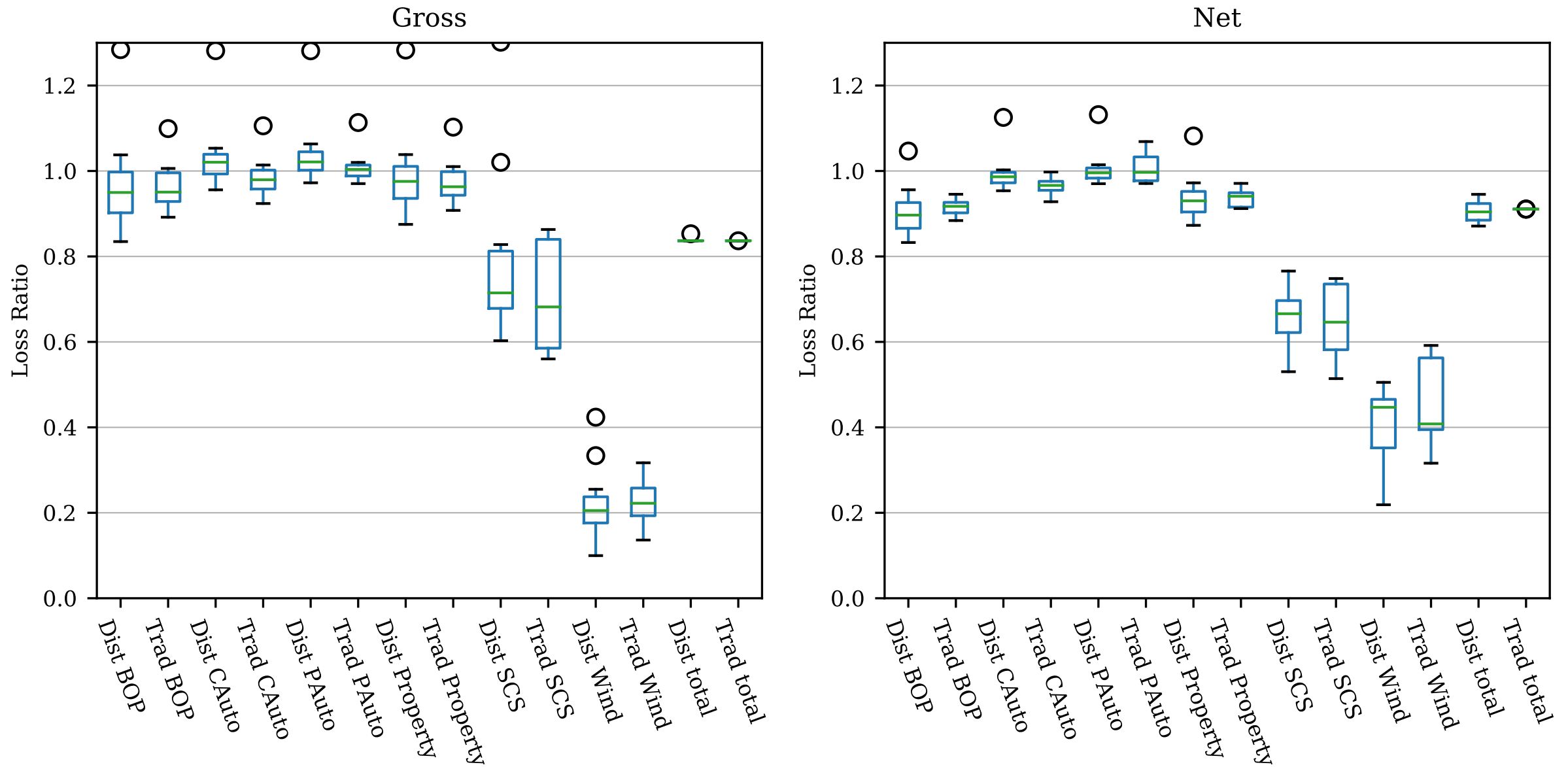


Figure 7: Broader ranges evident for distortion methods than standard methods. Tighter ranges for net than gross.

G.04. Conclusions and Answers

1. How does price vary with risk? With volatility vs. tail risk?

- Price and risk are related in a subtle and multifaceted way
- Total pool premium is a function of the amount of capital, typically determined by regulation, and the cost of capital, determined in the capital markets
- Different layers of capital have different costs, c.f., a credit yield curve
- In a multiline pool, relative volatility between lines determines the appropriate allocation between lines
- Line pricing depends on the relative consumption of low return period, high-cost equity capital and high return period, lower-cost capital

2. Can reasonable risk loads for non-cat lines be estimated from cat bond price data?

- Surprisingly, YES!
- Distortions calibrated to cat bond prices, typically with return periods of 10 years or more, produce very reasonable margins for the total portfolio and each for individual line, including low volatility lines such as personal auto
- The pure cost of insurance risk can be abstracted from cat bond market prices

3. Are negative risk loads ever appropriate? Why? When?

- Negative margins can be economically appropriate
- They reflect worse-than-stand-alone outcomes for low risk lines vs. high risk when they are combined in a limited liability pool
- They are a symptom of ineffective diversification or inadequate capital levels or both
- Negative margins become increasingly unlikely in effectively diversified and well-capitalized pools
- There will never be negative margins between independent lines if there is no chance of default

4. Should all lines target the same return on capital (ROE)?

- The economic cost of capital varies with probability of loss
- Different lines consume different amounts of capital by probability of loss
- Therefore target return by line should vary
- Low risk lines consume relatively little high, cost capital
- High risk lines consume relative more, low cost capital
- Offsetting dynamics create indeterminate impact on overall margin

5. Should target ROE be calibrated to gross or net returns? Are the two the same?

- Gross and net books have different economic costs of capital
- Economic premium represents the fair price of bearing risk against an optimal capital structure
- Net ROE targets generally higher(!) than gross because it is less volatile and consumes less cheap tail capital
- Optimal capital structure differs gross to net
 - $\text{Gross} = \text{Net} + \text{hedges and reinsurance}$
 - Techniques developed here also apply to pricing hedges

6. Can the same distortion be used to price gross and net? All lines?

- A distortion applies equally to all gross and net books
- Distortions calibrated to underlying economics, independent of capital structure
- One distortion can be used to price insurance supported by a range of asset levels
- A distortion produces risk-sensitive pricing; in particular gross and net ROEs generally differ
- Distortions calibrated to cat bond prices produce reasonable premiums, both gross and net

- Distortion pricing consistent with ambiguity averse risk-bearers
- Ambiguity varies by management team as well as line
- Ambiguity minimized by high-quality information signal from management, created by track-record of achieving plan
- Investors dislike volatility in results because it introduces ambiguity about quality of management
- Hard to disentangle ambiguity and risk

7. Is pricing in one line independent of reinsurance on other lines?

- The economic value to the insured varies with reinsurance on other lines
- The fair market value of liability payments will also vary
- It can be appropriate to vary targets for non-reinsured lines as a result of purchases elsewhere
- Reinsurance differs from use of debt capital, because it transfers risk