Pricing Insurance Risk

Module F: Cat Bonds, Their Pricing, and Its Implications for Pricing Non-Cat Lines

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Created 2021-01-19 09:14:46.499965

convex risk

independent | informed | imaginative

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Module F Contents

F.01. Catastrophe Bonds and Their Pricing

Catastrophe Bonds

Coverage, perils, geography, and triggers

- Cat bonds provide collateralized reinsurance for catastrophe perils such as hurricane, typhoons, earthquake, tornado-hail, severe convective storms, and winter storms
- The first cat bonds were issued in 1996
- Outstanding bonds cover US Eastcoast wind, US quake, Japanese wind and quake, and European wind and quake, and other perils
- Bonds issued on per occurrence and aggregate basis
- Coverage provided with indemnity or index loss trigger
- Total market has approximately USD 45 billion outstanding limit
- See Artemis for more information

Catastophe Bonds

Pricing statistics

- **EL** = expected loss, expressed as a percentage of limit; measures benefit from buyer's perspective
- **ROL** = rate on line, premium expressed as a percentage of limit; measures cost from buyer's perspective
- ROL also called the **spread**, the additional coupon paid by the sponsor to the bond holder (investment income on collateral passes back to the bond holder, the spread is paid in-addition)
- Bond loss ratio is EL / ROL

Factors that influence pricing

- Relationship between ROL and EL influenced by numerous factors, including: attachment and layer size, peril and geography (assessment of hazard and ability of models to estimate hazard), indemnity or index trigger, personal or commercial and insurance or reinsurance business, management track-record
- Our goal: model relationship between ROL and EL using a distortion function

Historical US Wind Cat Bond Pricing

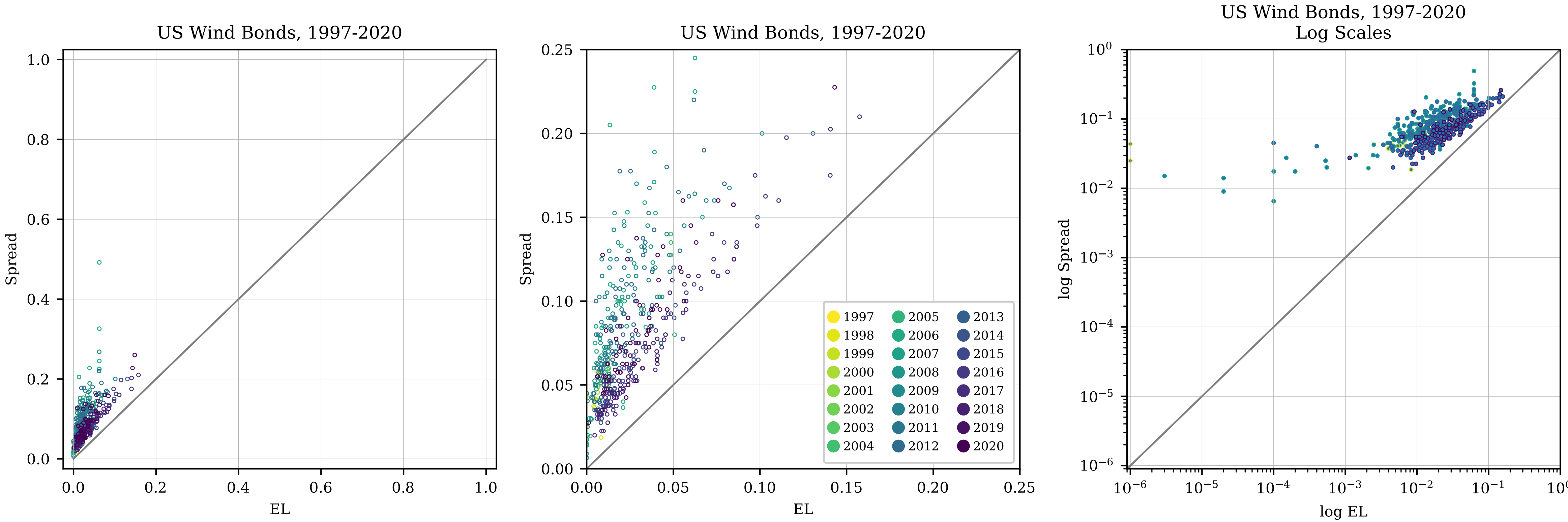


Figure 1: Spread (ROL) vs. EL on US wind (hurricane) exposed cat bonds since 1997. Color coding shows greater recent issuance. The left and middle plots differ only in scale. They show that cat bond data only includes observations for $s < 0.20$. The right hand plot is on a log scale, emphasizing highly rated (low default probability) bonds, and illustrating the well-known minimum-rate-on line phenomenon of reinsurance pricing. Data: Lane Financial LLC

Historical US Wind Cat Bond Pricing (2009-2020)

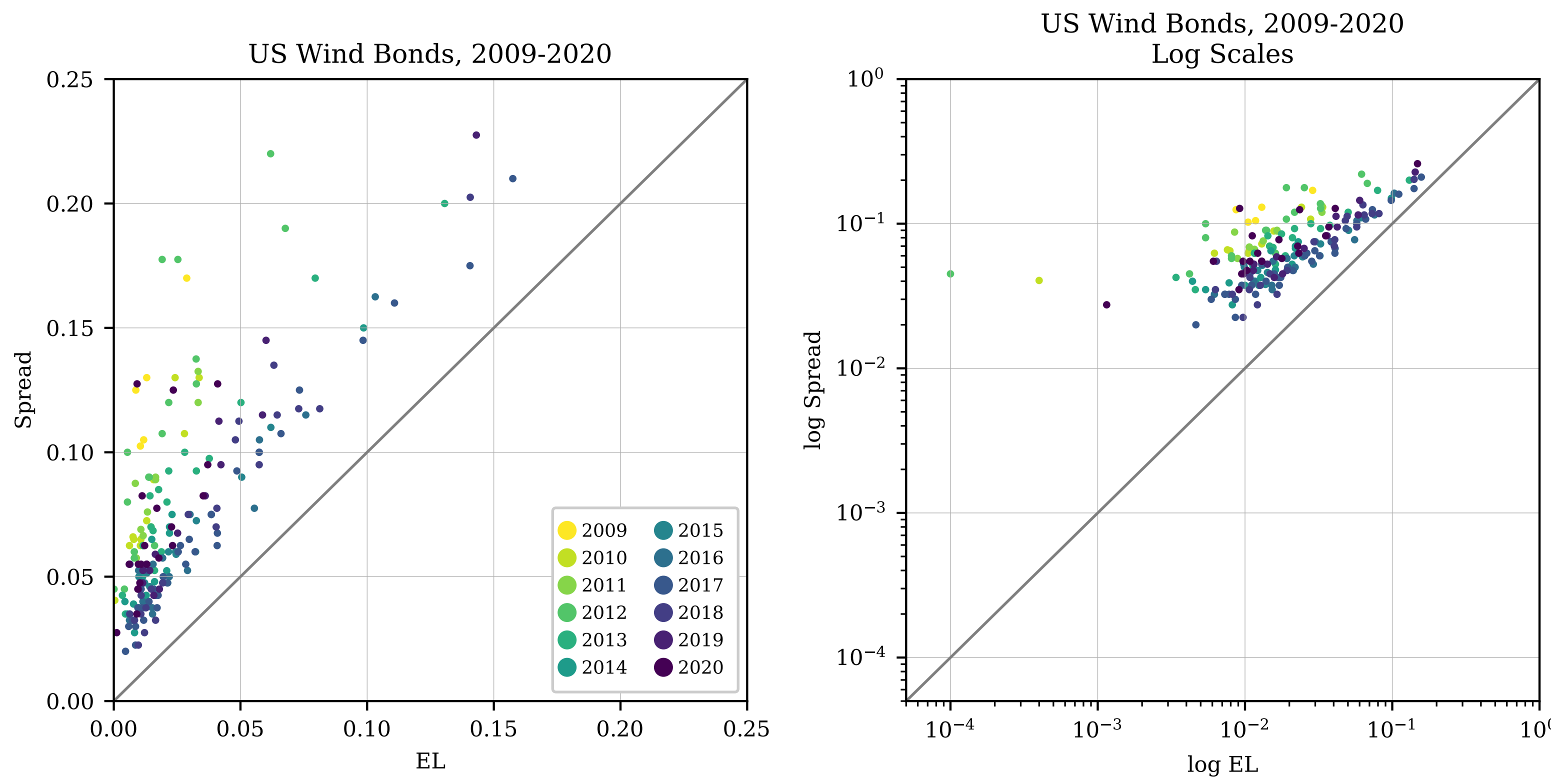


Figure 2: Spread (ROL) vs. EL on US wind (hurricane) exposed cat bonds, 2009-20. Left plot on linear scale and right on log scale. Data points are a subset of those shown on the previous figure. Data: Lane Financial LLC

Cat Bond Statistics, All Years

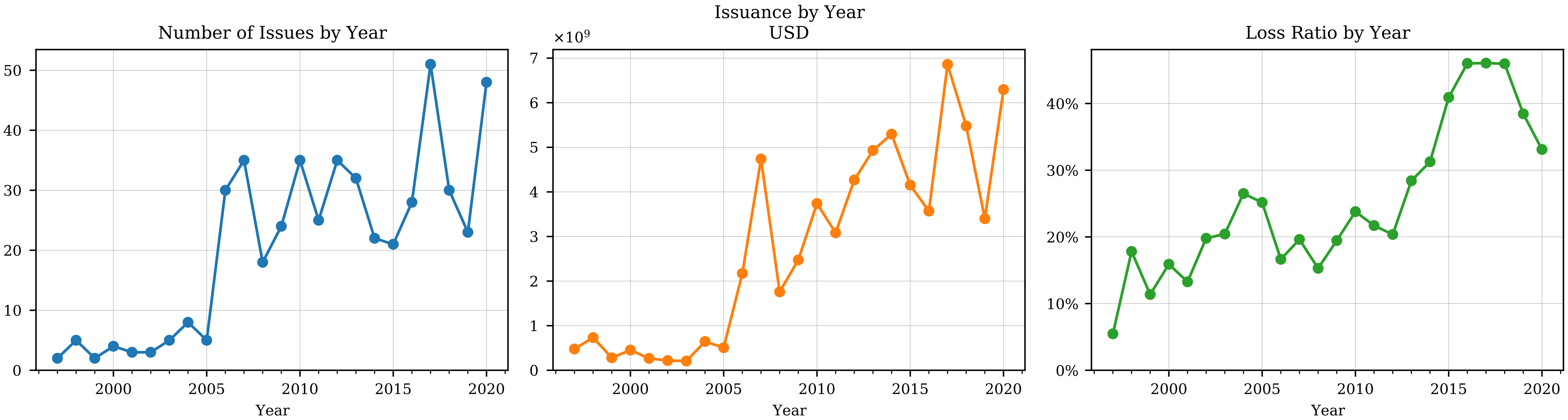


Figure 3: Number of issues, amount of issance, and average loss ratio by year, US wind exposed bonds only. A loss ratio of 0.2 indicates a premium equal to five times expected losses, etc. Data: Lane Financial LLC

Cat Bond Statistics, 2009-2020

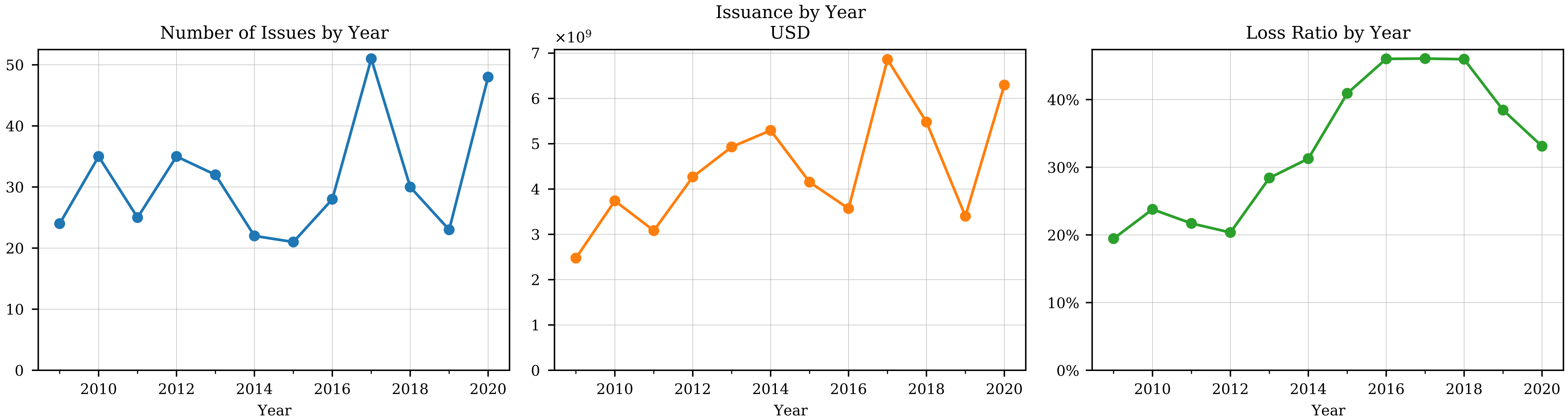


Figure 4: Number of issues, amount of issance, and average loss ratio by year, US wind exposed bonds only for more recent issues. Effect of 2017 storms on market clearly evident in decreasing loss ratios (increasing pricing). Data: Lane Financial LLC

F.02. Creating a Distortion From Cat Bond Prices in Theory

Creating a Distortion From Cat Bond Prices

Objective: use cat bond prices to calibrate a distortion that gives reasonable prices for all lines

- Reasonable: judged in relation to typical market loss ratios and historical results
- Why? Obviates need to select arbitrary pricing distortion
- **Creating** a distortion is not synonymous with **modeling** prices

Philosophies of modeling

- **Normative**: how should cat bonds be priced? Fitting a preferred parametric distortion with a theoretical justification
  - E.g., Wang transform replicates Black-Scholes option prices, under certain assumptions
- **Positive**: how are cat bonds priced? Smoothing, interpolate or extrapolate from observed prices
  - Smoothed pricing for similar bonds can be used as a predictive model
  - Approach preferred here
- **Predictive modeling**: estimate the price of an out of sample bond
- **Inferential statistics**: explain the price of an observed bond
- **Our goal**: something reaonable...

Creating a Distortion From Cat Bond Prices: Difficulties

Data limitations

- Data: set of observed market prices given by ELs and ROLs, i.e., $\{(s_i, g(s_i)) \mid i \in \mathrm{bonds}\}$, augmented with descriptive statistics about each issue
- **Problem**: Price data is concentrated in $s < 0.1$, a few points $0.1 < s < 0.2$, and generally none $s > 0.2$
- Leaves question of extrapolation
- A risk with $s > 0.2$ is equity-like; junk bond yields can reach 20% in stressed markets but historical default rates are lower
- Possible solution: linear interpolation from highest EL point $(s^*, g(s^*))$ to $(1, 1)$
  - Implies a constant ROE of $(g(s^*) - s^*)/(1 - g(s^*))$ over the range $s > s^*$
  - Constant return over $s > s^*$ justified by the fact that all layers of equity are fungible and earn the same return: there are no senior or subordinated common equity holders

Equity returns

- Data can be augmented by adding an equity-return point, prior to fitting
- Forces equity-like return for *volatility*, high-frequency losses, which often comports well with management preconceptions about investor preferences
- E.g., the point $(0.2, 0.3)$ corresponds to an ROE of $0.1/0.7 = 14$ percent

Creating a Distortion From Cat Bond Prices

Families of distortions

- There are numerous parametric families of distortions
  - Wang, dual, proportional hazard, TVaR, beta, etc.
- Many parametric families exhibit very high ROEs for low losses
- **Piecewise linear distortions** (PLD) are a simple, flexible and tractable alternative

Explore four approaches, all using PLDs

1. **Average of points**: fit a PLD to each data point $(s^*, g(s^*))$ and then average
2. **Convex envelope**: PLD fit through the extreme prices, connect-the-dots of the origin, the highest prices and $(1, 1)$
3. **Bagged convex envelope**: bootstrap aggregation, the average the convex envelopes determined by resampling subsets of the data to lowers the influence of extreme pricing points in the data
4. **Least squares** PLD fit with specified number of kink points; compute error on log scale to emphasize importance of fitting for small values of s

PLDs and each method described more fully in subseqent slides

Piecewise Linear Distortions (PLD)

A PLD is a weighted average of TVaRs at different probability levels

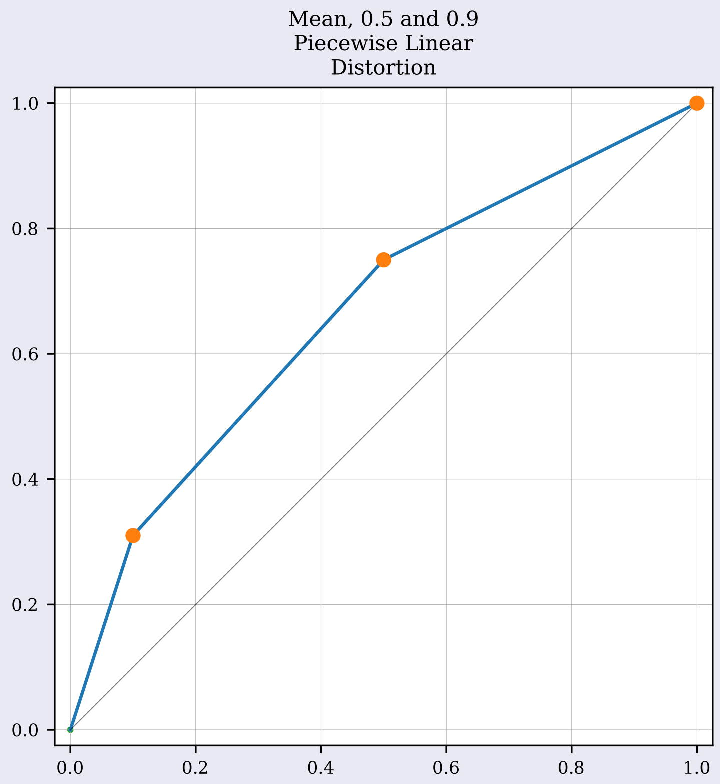


Figure 5: A piecewise linear distortion with kinks at 0.1, 0.5 and 1 and weights 0.2, 0.3 and 0.5. It corresponds to $0.5\text{TVaR}_0 + 0.3\text{TVaR}_{0.5} + 0.2\text{TVaR}_0.9$. $\text{TVaR}_{p=0}$ is the mean. The orange dots show the kink points, with values 0.31, 0.75, 1.

Advantages of PLDs

- Computationally simple
- Weights and kind points can be chosen arbitrarily
  - Kink at $s = 1$ ($p = 0$) adds a mean component, since 0-TVaR is the mean
  - Kink at $s = 0$ ($p = 1$) adds a minimum rate on line, since 1-TVaR is the maximum loss
- Constant returns between kink points
  - All equity (above last kink < 1) earns the same return
  - Maps to credit yield curve
- Any distortion can be approximated uniformly closely by a PLD
- Interpretation as a risk appetite statement comprised of multiple TVaR tolerances

Gotcha

- Distortions are based on exceedance probability: small s corresponds to large losses
- TVaR is based on the distribution function, with p close to 1 corresponding to large losses
- Thus PLD kink points are at $1 - p$ for p-TVaR

Point Distortions

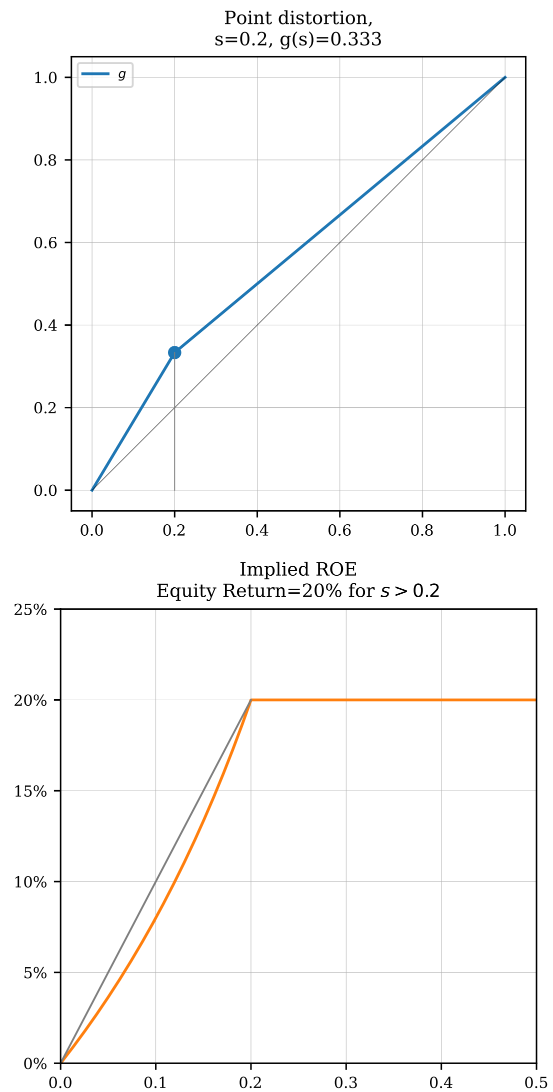


Figure 6: A point distortion defined by $(s, g(s)) = (0.2, 0.333)$ (top) and implied ROE=20% (bottom). The ROE is constant for $s > 0.2$ and sublinear from 0 to 0.2.

A point distortion (PD) is a PLD defined by a single price point

- A PD interpolates linearly between $(0,0)$, $(s, g(s))$, and $(1,1)$
- A PD is a weighted average of two TVaRs:
  - $p = (1-s)$-TVaR weighted $w = \dfrac{g(s)-s}{1-s}$
  - $p = 0$-TVaR (the mean) weigted $1 - w$
- The implied ROE at s and above is $\dfrac{g(s)-s}{1-g(s)}$
- The point with ROE equal to r at s has $g(s) = \dfrac{r}{1+r} + \dfrac{s}{1+r} = d + sv$

Average of Points Distortions

Average individual prices

- An **average of points distortion** is just the average of a number of PDs
  - Evaluate each PD at s
  - Average
- Smooths multiple observed prices
- Optionally: add an ROE-return point

index	EL	Spread	ROE
amin	0.0046	0.0225	0.0129
mean	0.039	0.0911	0.0548
amax	0.158	0.26	0.169

Figure 7: Summary of 40 data points used to create average of points distortion example.

index	EL	Spread	ROE
amin	0.0046	0.0225	0.0129
mean	0.039	0.0911	0.0548
amax	0.158	0.26	0.169

Figure 8: Summary of 40 data points used to create average of points distortion example.

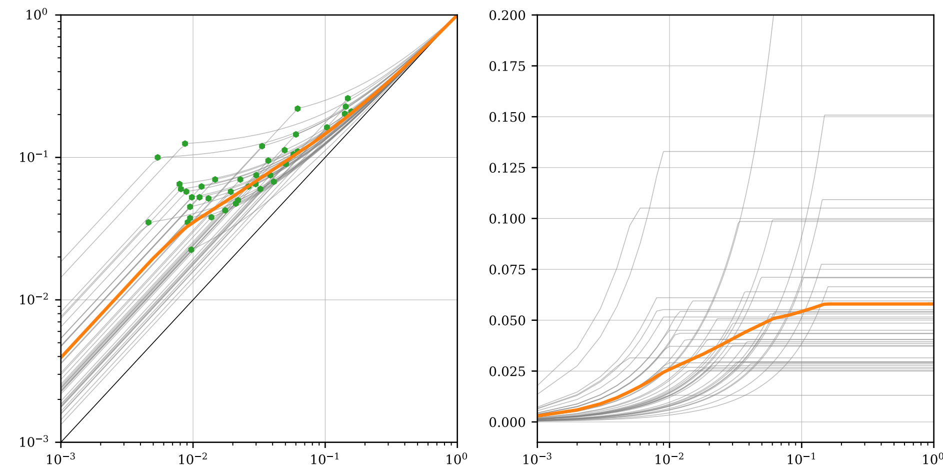


Figure 9: As above, log scales.

Convex Envelope Distortion

Convex envelope: join extreme points

- A **convex envelope distortion** (CED) is a PLD through the extreme points a given set of price observations
- Extreme points found as upper boundary of the convex hull of price points
- Example shows a CED through 40 price points
- Added point (purple hexagon) at $(s, g(s)) = (0.2, 0.36)$ enforces a 25% ROE for $s > 0.2$
- Thin lines show CED and ROE without the added ROE-point

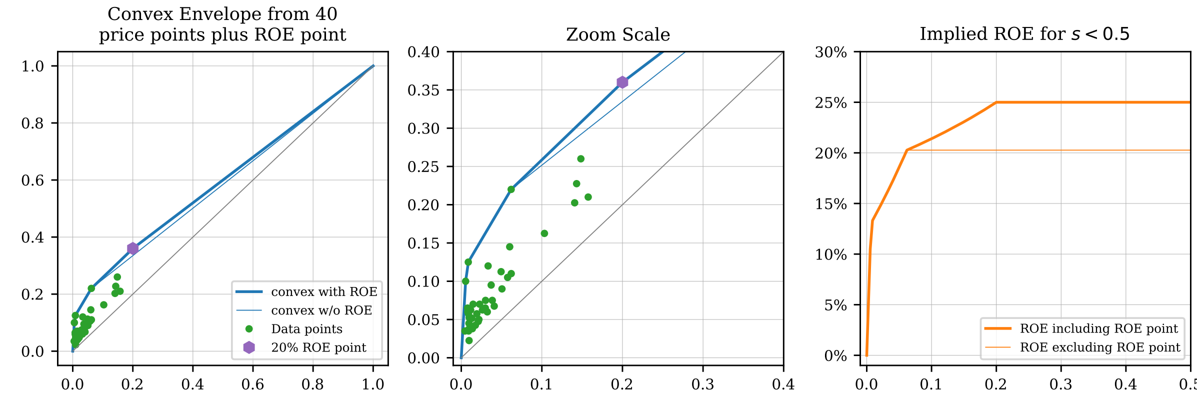


Figure 10: A convex envelope distortion defined by 40 observed pricing points plus a specified ROE point $(s, g(s)) = (0.2, 0.36)$ (left) and implied ROEs (right). The ROE is constant for $s > 0.2$ and sub-linear from 0 to 0.2.

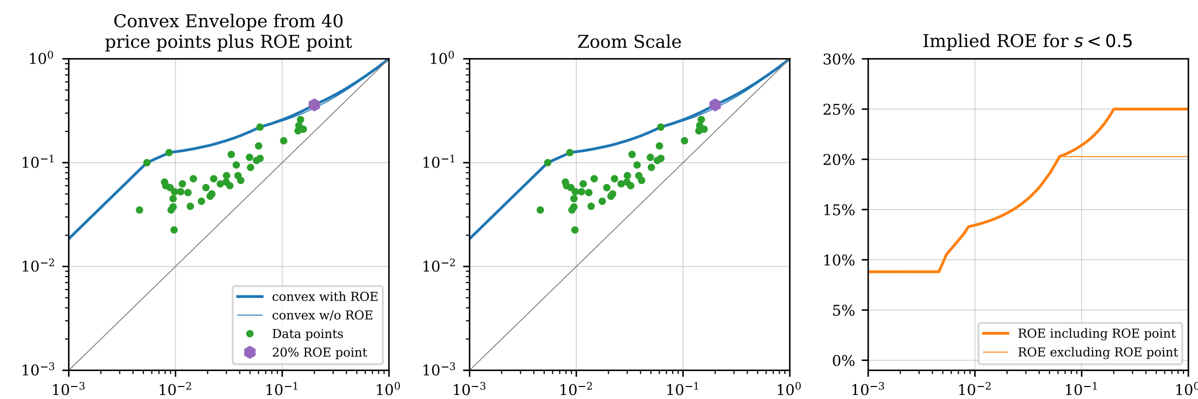


Figure 11: As above, on log scale.

Bagged Convex Envelope Distortion

Bootstrap Aggregation (bagging)

- The convex envelope represents a worst-case outcome, taking the maximum feasible price at each return period
- The convex envelope of a subset of pricing points would generally avoid extreme outcomes, providing a more balanced smoothing of data
- Bagging, as used in decision trees, simulates multiple samples of a fixed proportion of the data, fits a CED to each sample, and then averages the CEDs pointwise

- Example shows a bagged CED formed from 1000 resamples of 10 percent of the pricing data
- The proportion of points chosen controls interpolation betwen two extremes:
  - Taking 2.5 percent of data (i.e., one data point) method would reduce to the average of points
  - Taking 100 percent of the data gives the CED

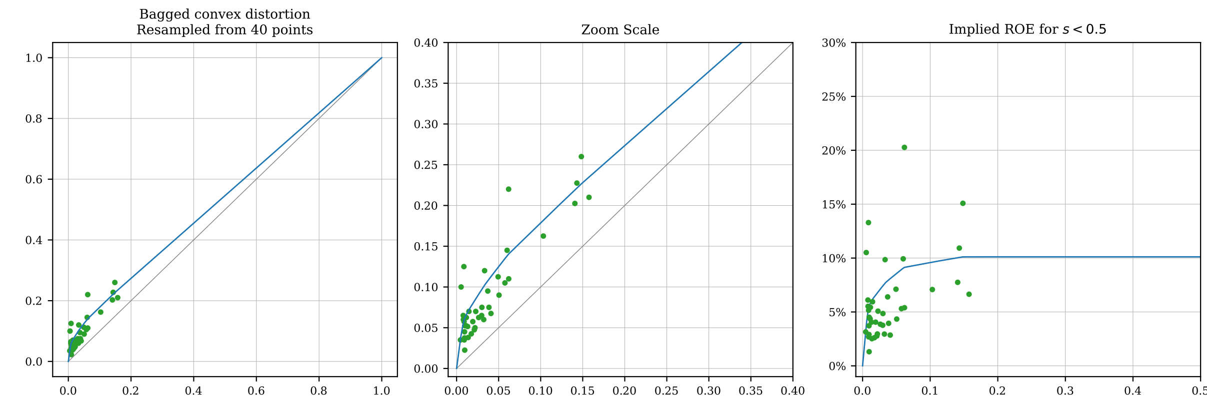


Figure 12: A bagged (bootstrap aggregated) convex envelope distortion defined as the average of 1000 resamples of 10 percent of 40 observed pricing points (left and middle, zoom) and implied ROEs (right).

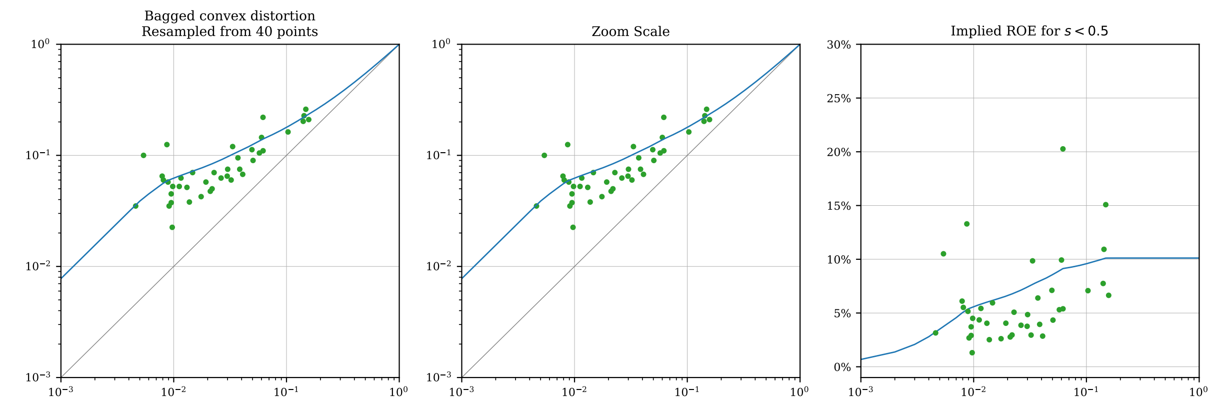


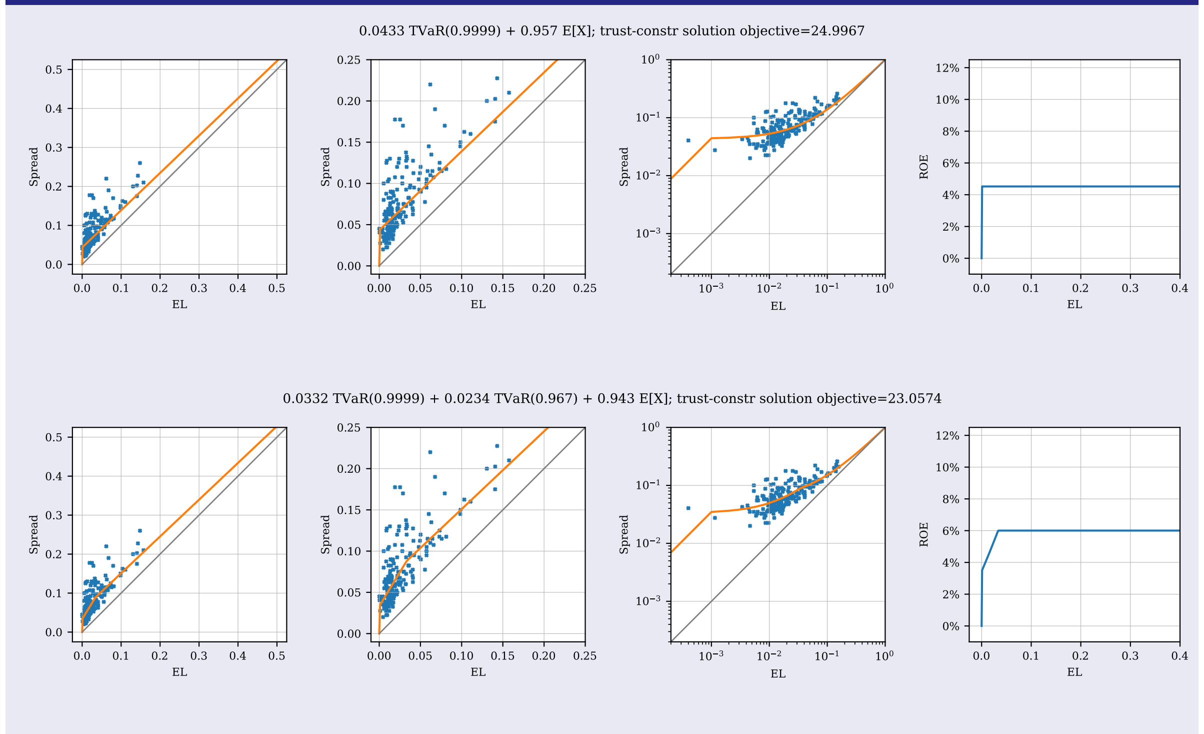
Figure 13: As above, on log scale.

Regression Fits

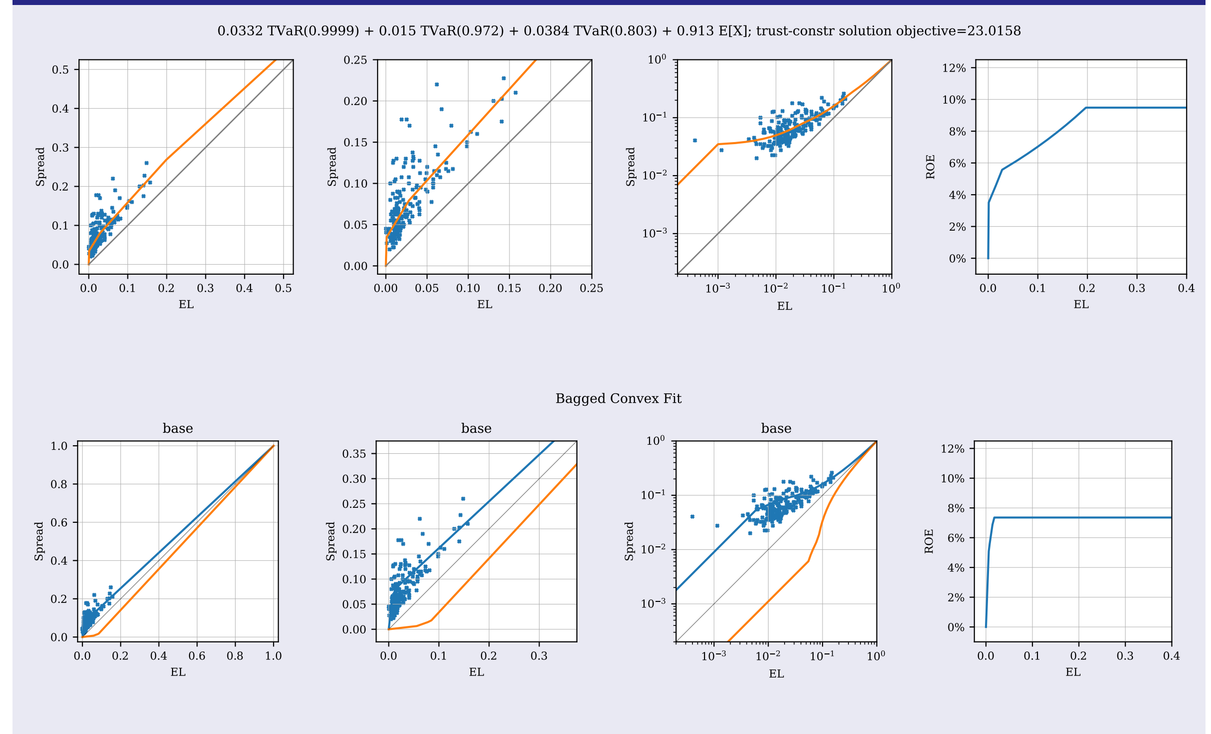
- Use least squares to fit a PLD with fixed number of kink points as a meta parameter
- To produce an equivalent measure, suitable for pricing, the PLD must weight the mean ($p = 0$) TVaR component
- Standard regression techniques dictate need to normalize variance before computing error
  - Working on a log-log scale appears reasonable
  - Error is $\log(\hat{g}(s)) - \log(g(s))$ rather than $\hat{g}(s) - g(s)$
- Consider three models
  1. One TVaR (kink) $p > 0$ plus the mean (two parameters)
  2. Two TVaRs $p_1, p_2 > 0$ plus the mean (four parameters)
  3. Three TVaRs $p_1, p_2, p_3 > 0$ plus the mean (six parameters)

Regression Fits

One TVaR; Two TVaRs plus mean



Three TVaRs plus mean; bagged convex



- Least squares fits to one-, two-, and three-TVaR plus mean models for cat bond pricing
  - From left: distortion and data, zoom of distortion, log scale distortion, implied ROE vs EL (right)
- Solutions all include minimum rate on line component with p very close to 1:
  - 0.0433 TVaR(0.9999) + 0.957 E[X]
  - 0.0332 TVaR(0.9999) + 0.0234 TVaR(0.967) + 0.943 E[X]
  - 0.0332 TVaR(0.9999) + 0.015 TVaR(0.972) + 0.0384 TVaR(0.803) + 0.913 E[X]
- Bagged convex fit shown (lower right) for comparison

F.03. Creating a Distortion From Cat Bond Prices in Practice

How Should General Insurance Market Prices Relate to Cat Bond Market Prices?

Cat bond market prices

- Very clearly defined hazard and peril
- Modeled: independent estimate of loss cost, no need to trust management
- Short tailed, though collateral lock-ups an issue post-2017
- Low frequency
- Alternative, zero-beta asset class

General insurance market prices

- Opaque, uncertain coverages
- No independent models for investors: must trust management
- Long tailed, capital provided on a permanent (equity) or semi-permanent (debt) basis
- Range of frequencies
- Locked up in stock insurer: subject to market systemic risks

General insurance markets characterized by high **uncertainty**

Base Distortion

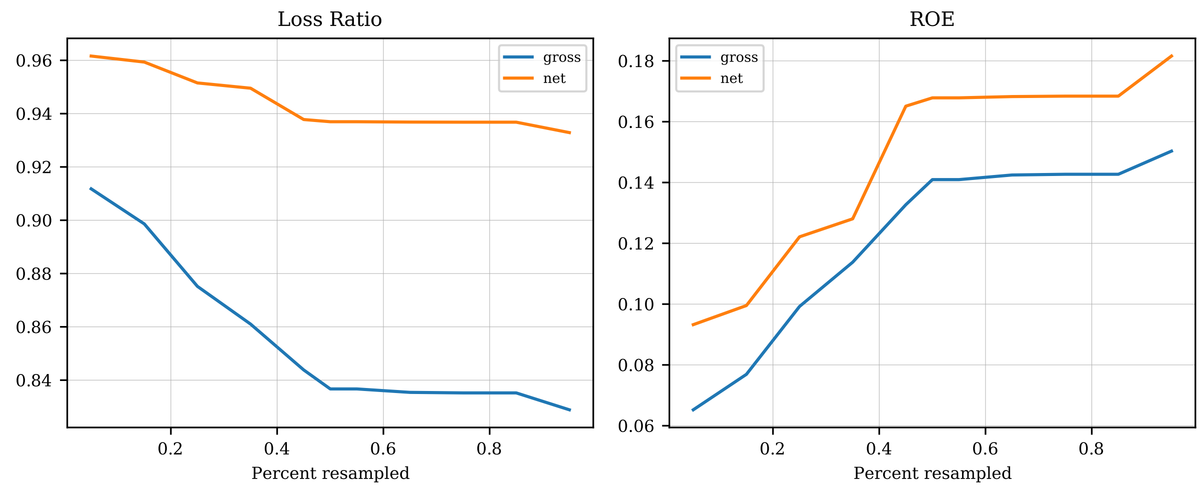


Figure 14: A BCE distortion is controlled by the percentage of prices resampled. The figure shows minimal impact above $p = 0.5$ Greater impact on more volatile gross portfolio than net, as expected.

Base distortion: bagged convex envelope

- Percent of prices re-sampled hyperparameter controls pricing
  - Resampling all points gives the convex envelope of all prices
  - Resampling one point gives the average of points
- Uncertainty considerations suggest pricing closer to convex envelope than average
  - Investors and underwriters tend to *think the worst*
- Hyperparameter profile (above) suggests $p = 0.5$
  - Gross loss ratio 83.7% and ROE 14.1%
  - Net loss ratio 93.7% and ROE 16.8%
- $p = 0.5$ BCE created with 1000 re-samples acts as reference for all future modeling
- Referred to as base in figures and tables
- Illustrated on next slide

Base Bagged Convex Envelope Distortion

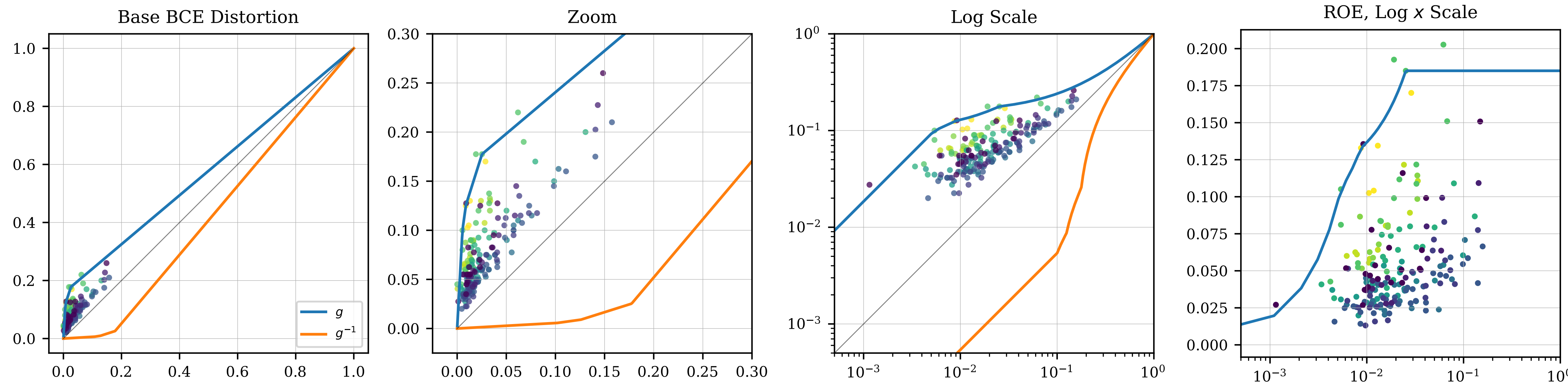


Figure 15: Distortion (blue), inverse (organge) and , and parameterizing points (color coded by year). 1000 re-samples of $p = 0.5$ proportion of 202 US wind exposed cat bond prices, 2009 to 2020. Third plot shown on a log scale. Right hand plot show ROE vs. log exceedance probability. Constant ROE above last kink, of 18.5%. Going forward base is used as the reference distortion to calibrate all other methods.

Distortions Calibrated to Cat Bond Pricing

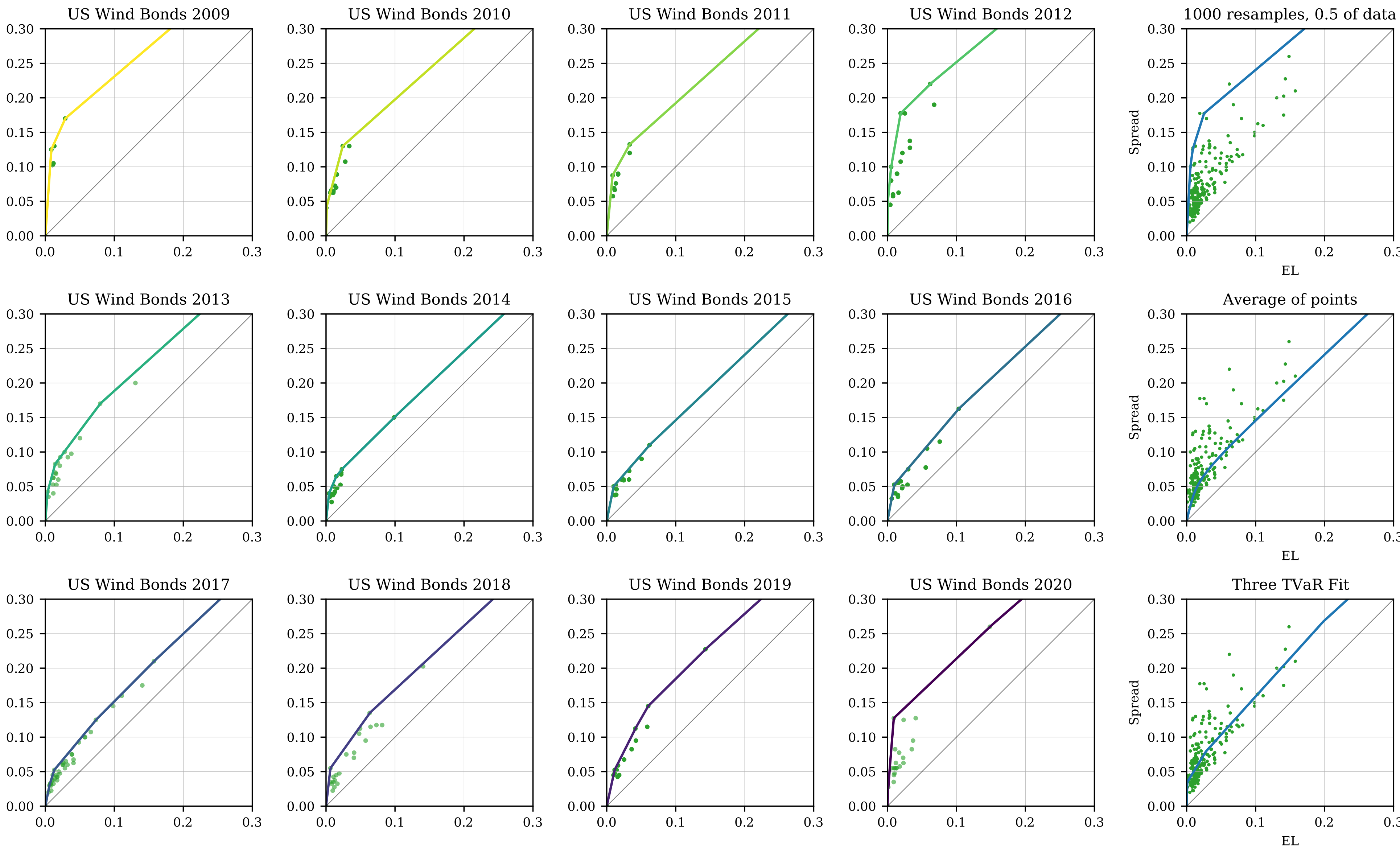


Figure 16: Convex envelope distortions calibrated to US wind exposed cat bonds by issue year (left 12 plots) plus the base bagged convex distortion, the average of points, and the three-TVaR regression fit.

Distortions Calibrated to Cat Bond Pricing, Log Scale

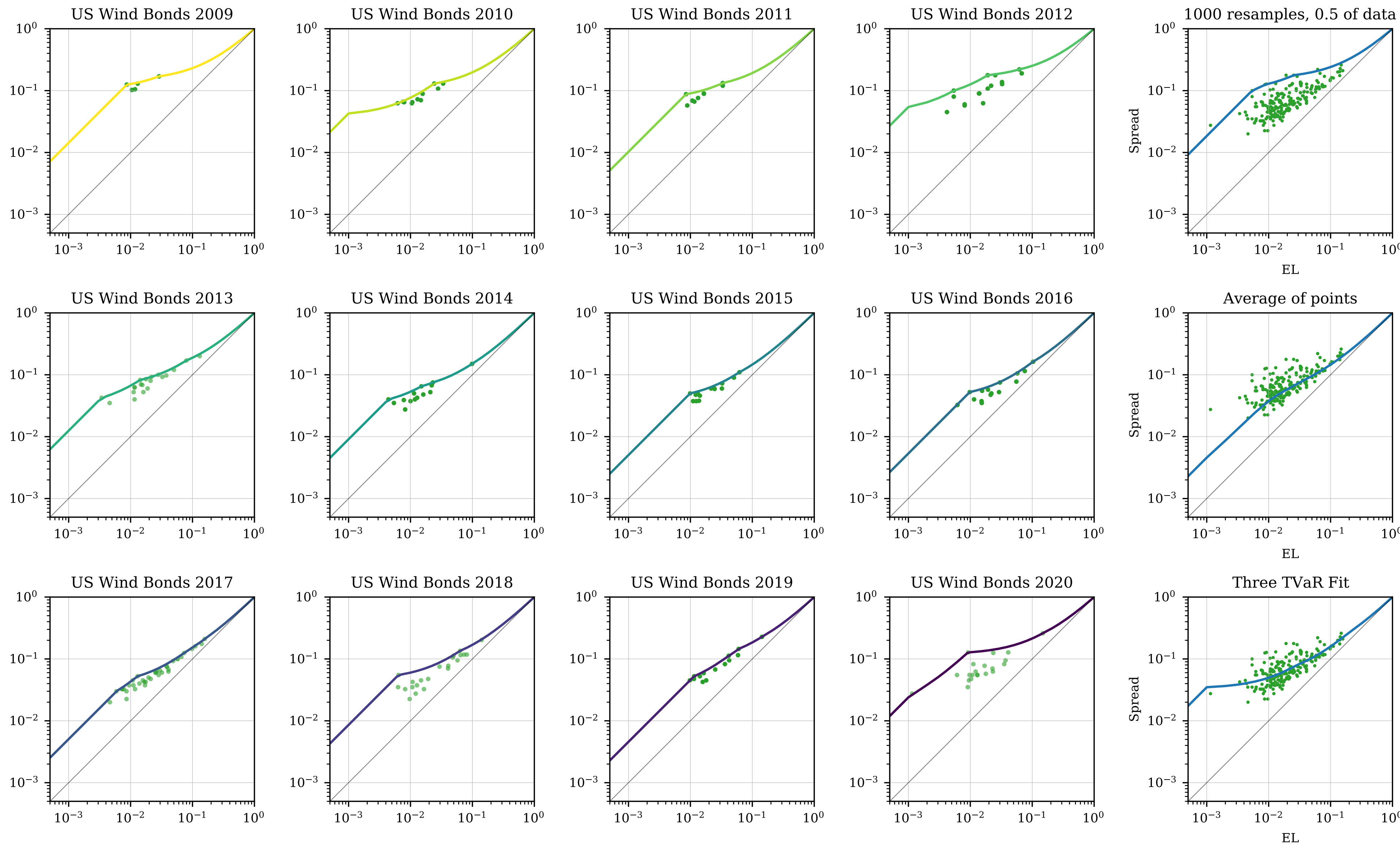


Figure 17: Distortions from previous slide on a log-log scale. The minimum rate on line behavior is more evident.

Gross Portfolio Statistics by Cat Bond Distortion, 2009 to 2020

stat	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	average	base	gross roe	regression
Expected Loss	99,175	99,175	99,175	99,175	99,175	99,175	99,175	99,175	99,175	99,175	99,175	99,175	99,175	99,175	99,175	99,175
Loss Ratio	0.849	0.889	0.891	0.831	0.907	0.932	0.946	0.94	0.945	0.922	0.931	0.857	0.952	0.837	0.837	0.93
Margin	17,578	12,339	12,168	20,238	10,141	7,224	5,705	6,371	5,767	8,431	7,318	16,532	4,951	19,355	19,355	7,505
Premium	116,753	111,514	111,343	119,412	109,315	106,398	104,880	105,545	104,941	107,605	106,492	115,706	104,125	118,530	118,530	106,680
Leverage	0.839	0.773	0.77	0.875	0.746	0.712	0.695	0.702	0.695	0.726	0.713	0.826	0.686	0.863	0.863	0.715
Surplus	139,097	144,336	144,507	136,438	146,535	149,452	150,970	150,305	150,909	148,245	149,358	140,144	151,725	137,320	137,320	149,170
ROE	0.126	0.0855	0.0842	0.148	0.0692	0.0483	0.0378	0.0424	0.0382	0.0569	0.049	0.118	0.0326	0.141	0.141	0.0503
Assets	255,850	255,850	255,850	255,850	255,850	255,850	255,850	255,850	255,850	255,850	255,850	255,850	255,850	255,850	255,850	255,850

Table 1: Gross portfolio statistics by cat bond distortion, 2009-2015. 0.996 VaR capital standard. Loss ratios (interpreted as combineds with no expenses) fall into reasonable range.

Net Portfolio Statistics by Cat Bond Distortion, 2009 to 2020

stat	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	average	base	net roe	regression
Expected Loss	97,405	97,405	97,405	97,405	97,405	97,405	97,405	97,405	97,405	97,405	97,405	97,405	97,405	97,405	97,405	97,405
Loss Ratio	0.941	0.957	0.958	0.933	0.963	0.976	0.979	0.976	0.978	0.97	0.967	0.947	0.98	0.937	0.937	0.972
Margin	6,071	4,404	4,243	6,973	3,737	2,385	2,050	2,428	2,241	2,995	3,310	5,472	1,942	6,553	6,553	2,842
Premium	103,476	101,809	101,648	104,378	101,142	99,790	99,455	99,833	99,646	100,400	100,715	102,877	99,347	103,958	103,958	100,247
Leverage	2.62	2.47	2.46	2.7	2.42	2.31	2.28	2.31	2.3	2.36	2.38	2.56	2.28	2.66	2.66	2.34
Surplus	39,524	41,191	41,352	38,622	41,858	43,210	43,545	43,167	43,354	42,600	42,285	40,123	43,653	39,042	39,042	42,753
ROE	0.154	0.107	0.103	0.181	0.0893	0.0552	0.0471	0.0563	0.0517	0.0703	0.0783	0.136	0.0445	0.168	0.168	0.0665
Assets	143,000	143,000	143,000	143,000	143,000	143,000	143,000	143,000	143,000	143,000	143,000	143,000	143,000	143,000	143,000	143,000

Table 2: Net portfolio statistics by cat bond distortion, 2009-2020. 0.996 VaR capital standard, applied to net losses. ROE increases: the net book uses fewer assets more intensively, but the dollars of margin decrease. Since dollars of margin decrease, the loss ratios increase. The cost to the insured includes the cost of reinsurance which is not accounted for in this Table.

Gross and Net Loss Ratio by Cat Bond Distortion

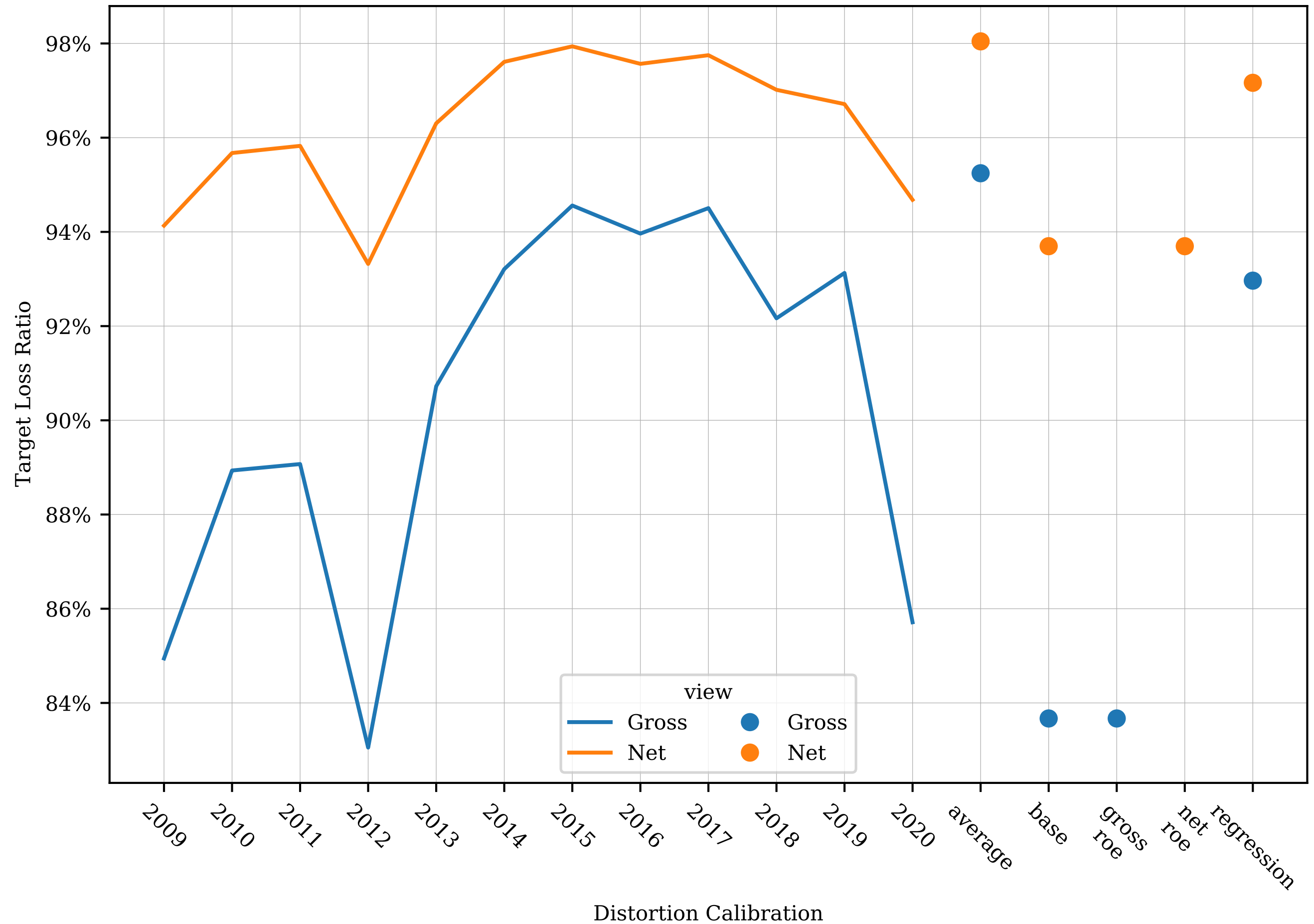


Figure 18: Gross and net total loss ratios by cat bond-calibrated distortion. Softening cat reinsurance market between 2012 and 2017 evident in increasing loss ratios. Total portfolio loss ratios are reasonable, even though underlying distortions are only calibrated to catastrophe bond prices. Market combined ratios, reflecting expenses, are around 5 points higher, see **??**.

Gross and Net Loss Ratio and Roe by Cat Bond Distortion

Calibration	Gross		Net	
	Loss Ratio	ROE	Loss Ratio	ROE
2009	0.849	0.126	0.941	0.154
2010	0.889	0.0855	0.957	0.107
2011	0.891	0.0842	0.958	0.103
2012	0.831	0.148	0.933	0.181
2013	0.907	0.0692	0.963	0.0893
2014	0.932	0.0483	0.976	0.0552
2015	0.946	0.0378	0.979	0.0471
2016	0.94	0.0424	0.976	0.0563
2017	0.945	0.0382	0.978	0.0517
2018	0.922	0.0569	0.97	0.0703
2019	0.931	0.049	0.967	0.0783
2020	0.857	0.118	0.947	0.136
average	0.952	0.0326	0.98	0.0445
base	0.837	0.141	0.937	0.168
gross roe	0.837	0.141	NaN	NaN
net roe	NaN	NaN	0.937	0.168
regression	0.93	0.0503	0.972	0.0665
Min	0.831	0.0326	0.933	0.0445
Average	0.9	0.0793	0.961	0.0984
Max	0.952	0.148	0.98	0.181
Range	0.122	0.116	0.0473	0.136

Table 3: Total portfolio loss ratio and ROE calibrated using prices for US Wind-exposed, indemnity cat bonds, from different time periods.

Pricing Non-Cat Business Using Cat Bond Data

- Table 3 shows distortions calibrated using cat bond data produce reasonable underwriting margins for the total portfolio
  - Expenses excluded; loss ratio can be considered a combined ratio
  - See **??**
- Next two exhibits show the gross and net implied loss ratios by distortion by line
  - Lines considered on a stand-alone basis
  - 0.996 VaR capital
  - sop shows sum-of-parts weighted loss ratio
  - Non-cat lines are unchanged gross vs. net and are only shown for gross
  - Loss ratios are reasonable
  - Market cycle more evident in high risk lines than low

Gross and Net Loss Ratio by Line and Distortion on Stand-Alone Basis

distortion	BOP	CAuto	PAuto	Property	SCS	Wind	sop	total	Net SCS	Net Wind	Net sop	Net total
2009	0.865	0.904	0.963	0.885	0.467	0.197	0.762	0.849	0.566	0.402	0.86	0.941
2010	0.898	0.929	0.973	0.913	0.554	0.262	0.82	0.889	0.641	0.479	0.894	0.957
2011	0.901	0.931	0.974	0.916	0.558	0.263	0.822	0.891	0.652	0.489	0.897	0.958
2012	0.848	0.891	0.958	0.87	0.434	0.177	0.736	0.831	0.532	0.369	0.842	0.933
2013	0.912	0.938	0.977	0.925	0.602	0.304	0.847	0.907	0.687	0.522	0.909	0.963
2014	0.942	0.96	0.985	0.951	0.685	0.374	0.888	0.932	0.77	0.632	0.94	0.976
2015	0.95	0.965	0.987	0.958	0.728	0.434	0.907	0.946	0.797	0.665	0.948	0.979
2016	0.941	0.959	0.985	0.95	0.705	0.412	0.897	0.94	0.774	0.632	0.939	0.976
2017	0.945	0.962	0.986	0.954	0.725	0.439	0.906	0.945	0.788	0.651	0.944	0.978
2018	0.928	0.95	0.981	0.939	0.647	0.342	0.87	0.922	0.732	0.572	0.925	0.97
2019	0.921	0.945	0.979	0.933	0.665	0.392	0.88	0.931	0.717	0.548	0.918	0.967
2020	0.876	0.913	0.966	0.895	0.486	0.206	0.775	0.857	0.595	0.432	0.872	0.947
average	0.952	0.967	0.988	0.96	0.752	0.475	0.917	0.952	0.806	0.677	0.95	0.98
base	0.855	0.897	0.96	0.877	0.445	0.182	0.745	0.837	0.547	0.383	0.85	0.937
gross roe	0.873	0.911	0.966	0.892	0.456	0.179	0.752	0.837	0.586	0.419	0.869	0.945
net roe	0.856	0.898	0.961	0.877	0.419	0.158	0.722	0.815	0.549	0.383	0.85	0.937
regression	0.931	0.952	0.982	0.942	0.675	0.377	0.882	0.93	0.751	0.606	0.931	0.972
Min	0.848	0.891	0.958	0.87	0.419	0.158	0.722	0.815	0.532	0.369	0.842	0.933
Average	0.902	0.931	0.974	0.917	0.579	0.296	0.825	0.89	0.668	0.513	0.899	0.958
Max	0.952	0.967	0.988	0.96	0.752	0.475	0.917	0.952	0.806	0.677	0.95	0.98
Range	0.105	0.0755	0.0301	0.0902	0.333	0.317	0.195	0.137	0.274	0.308	0.108	0.0473

Table 4: Gross and net loss ratio by line and distortion on stand-alone basis. `SCS` and `Wind` subjet to per occurrence reinsurance, lowering net event total to 25M, which has a dramatic effect on line and total loss ratios. `sop` denotes the sum of parts, whereas `total` is the diversified total.

Gross and Net ROE by Line and Distortion on Stand-Alone Basis

distortion	BOP	CAuto	PAuto	Property	SCS	Wind	sop	total	Net SCS	Net Wind	Net sop	Net total
2009	0.154	0.155	0.156	0.154	0.134	0.123	0.132	0.126	0.154	0.153	0.154	0.154
2010	0.107	0.108	0.109	0.108	0.0909	0.0821	0.0896	0.0855	0.108	0.107	0.108	0.107
2011	0.103	0.103	0.105	0.103	0.0893	0.0817	0.0881	0.0842	0.103	0.103	0.103	0.103
2012	0.181	0.182	0.184	0.182	0.157	0.143	0.155	0.148	0.182	0.18	0.181	0.181
2013	0.0897	0.0905	0.0921	0.0902	0.0734	0.0655	0.0727	0.0692	0.0866	0.089	0.0892	0.0893
2014	0.0553	0.0555	0.056	0.0554	0.05	0.0472	0.0496	0.0483	0.055	0.0547	0.0552	0.0552
2015	0.0472	0.0475	0.0481	0.0474	0.0402	0.0364	0.0396	0.0378	0.0465	0.047	0.0471	0.0471
2016	0.0565	0.0571	0.0583	0.0569	0.0453	0.0399	0.0448	0.0424	0.0538	0.0546	0.0557	0.0563
2017	0.052	0.0525	0.0537	0.0523	0.041	0.0357	0.0405	0.0382	0.0494	0.0503	0.0512	0.0517
2018	0.0706	0.0711	0.0722	0.0709	0.0599	0.0546	0.0594	0.0569	0.0684	0.0713	0.0705	0.0703
2019	0.0789	0.0801	0.0826	0.0796	0.055	0.0436	0.054	0.049	0.074	0.0795	0.0785	0.0783
2020	0.137	0.137	0.139	0.137	0.123	0.115	0.122	0.118	0.135	0.133	0.136	0.136
average	0.0447	0.0451	0.046	0.0449	0.0355	0.0306	0.0349	0.0326	0.044	0.0445	0.0447	0.0445
base	0.168	0.169	0.171	0.169	0.148	0.137	0.146	0.141	0.169	0.167	0.169	0.168
gross roe	0.141	0.141	0.141	0.141	0.141	0.141	0.141	0.141	0.141	0.141	0.141	0.141
net roe	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.168
regression	0.067	0.0679	0.0697	0.0675	0.0525	0.0465	0.0525	0.0503	0.0613	0.0614	0.0647	0.0665
Min	0.0447	0.0451	0.046	0.0449	0.0355	0.0306	0.0349	0.0326	0.044	0.0445	0.0447	0.0445
Average	0.0981	0.0986	0.0998	0.0984	0.0855	0.079	0.0847	0.0816	0.0968	0.0971	0.0978	0.0978
Max	0.181	0.182	0.184	0.182	0.168	0.168	0.168	0.168	0.182	0.18	0.181	0.181
Range	0.136	0.137	0.138	0.137	0.132	0.137	0.133	0.135	0.138	0.135	0.137	0.136

Table 5: Gross and net ROE by line and distortion on stand-alone basis. `SCS` and `Wind` subjet to per occurrence reinsurance, lowering net event total to 25M, which has a dramatic effect on line and total loss ratios. `sop` denotes the sum of parts, whereas `total` is the diversified total.

Return Discount vs Exceeding Probability for Cat Bond Distortions

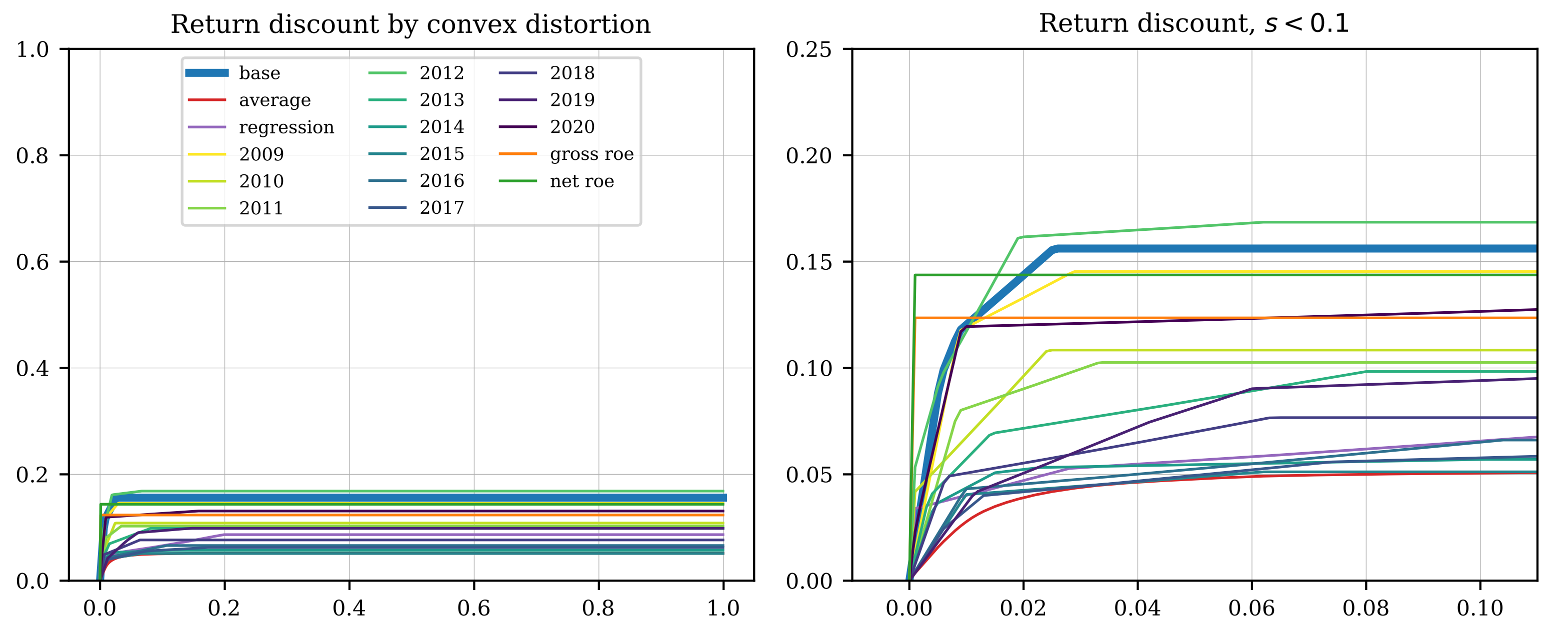


Figure 19: Return discount vs. exceeding probability for the cat bond calibrated distortions. Return discount $d = r/(1 + r)$ is preferred to ROE r because $0 \le d \le 1$, whereas $r \to \infty$ is possible. For small r, d is slightly less than r. All distortions exhibit a "maximium ROE" from 5 to 17 percent—a reasonable range compared to actual insurer ROEs over the period. **??** shows analogous plot for parametric distortions.