Pricing Insurance Risk

Module E: Distortions: Definition, Examples, and Properties

Stephen J. Mildenhall

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convex risk

independent | informed | imaginative

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Module E Contents

Summary

- Define and introduce distortions
- Introduce catastrophe (cat) bonds and historical cat bond pricing data
- Investigate ways to create distortions from cat bond pricing data, more smoothing than modeling
- Implied loss ratios on gross and net model portfolio, and stand-alone by line, by distortion compared to typical market loss ratios
- Select [one or two] distortions to carryover to Part C for more detailed allocation analysis

E.01. Distortions and Distortion Pricing Operators

Distortions

Distortion functions

- A **distortion function** relates probability of loss to premium or economic value
- It prices a binary 0/1 risk with different probabilities of loss
  - For a binary risk, the probability of loss equals the expected loss (EL) and the premium equals the rate on line
- Formally, a distortion is a function $g : [0,1] \to [0,1]$ satisfying
  1. $g(0) = 0, g(1) = 1$
  2. g increasing
  3. g concave (bowed down)

Universal properties

- Distortions are always
  - Continuous, except possibly at 0
  - Differentiable almost everywhere
  - Lie above the diagonal, $g(s) > s$ implying a non-negative margin
- Examples: proportional hazard, dual moment, Wang transform, TVaR
- General fact: all distortions are weighted avereages of TVaRs

Distortions

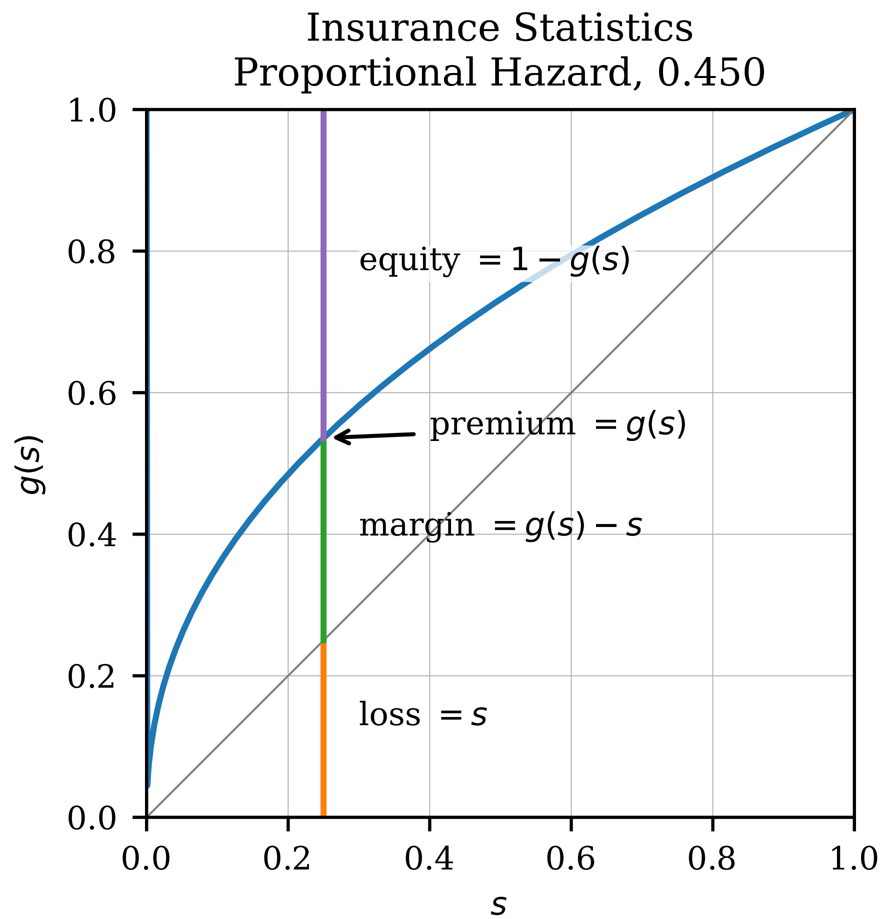

Figure 1: Illustration of a proportional hazard distortion, showing the loss, margin, premium and equity componets for a given s. Given g the layer loss ratio is $s/g(s)$, layer ROE is $(g(s) - s)/(1 - g(s))$ and layer leverage $g(s)/(1 - g(s))$.

Distortion Pricing Operator

- Distortions determine pricing operator; it is given as a re-weighted sum of outcomes

$$\rho_g(X) = \int_0^\infty g(S(x))dx = \int_0^\infty xg'(S(x))f(x)dx$$

  where $X \geq 0$ is a loss random variable with survival function S and density f

- ρ_g can be applied to limited losses or unlimited losses

- Properties of g correspond to properties of ρ_g
  - $g(0) = 0, g(1) = 1$ ensure ρ_g is translation invariant
  - g must be increasing to ensure ρ_g is monotonic
  - g must be concave to ensures ρ_g is sub-additive

Astin / Cas

- Properties of β
- ASTIN twelve plot view