

# Pricing Insurance Risk

## Module B: Market Assumptions

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independent | informed | imaginative

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## **B.01. Market Assumptions**

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# All Models Are Wrong—Some Are Useful

## Objective: build a useful simple model

A coherent insurance pricing framework starts with a simple but realistic model

Make four simplifying assumptions: three benign and one tricky

### 1. No expenses

- Risk premium can be grossed-up for expenses

### 2. No investment income

- Work with present value losses
- Credit at embedded yield: insured exposed to asset performance through possible default

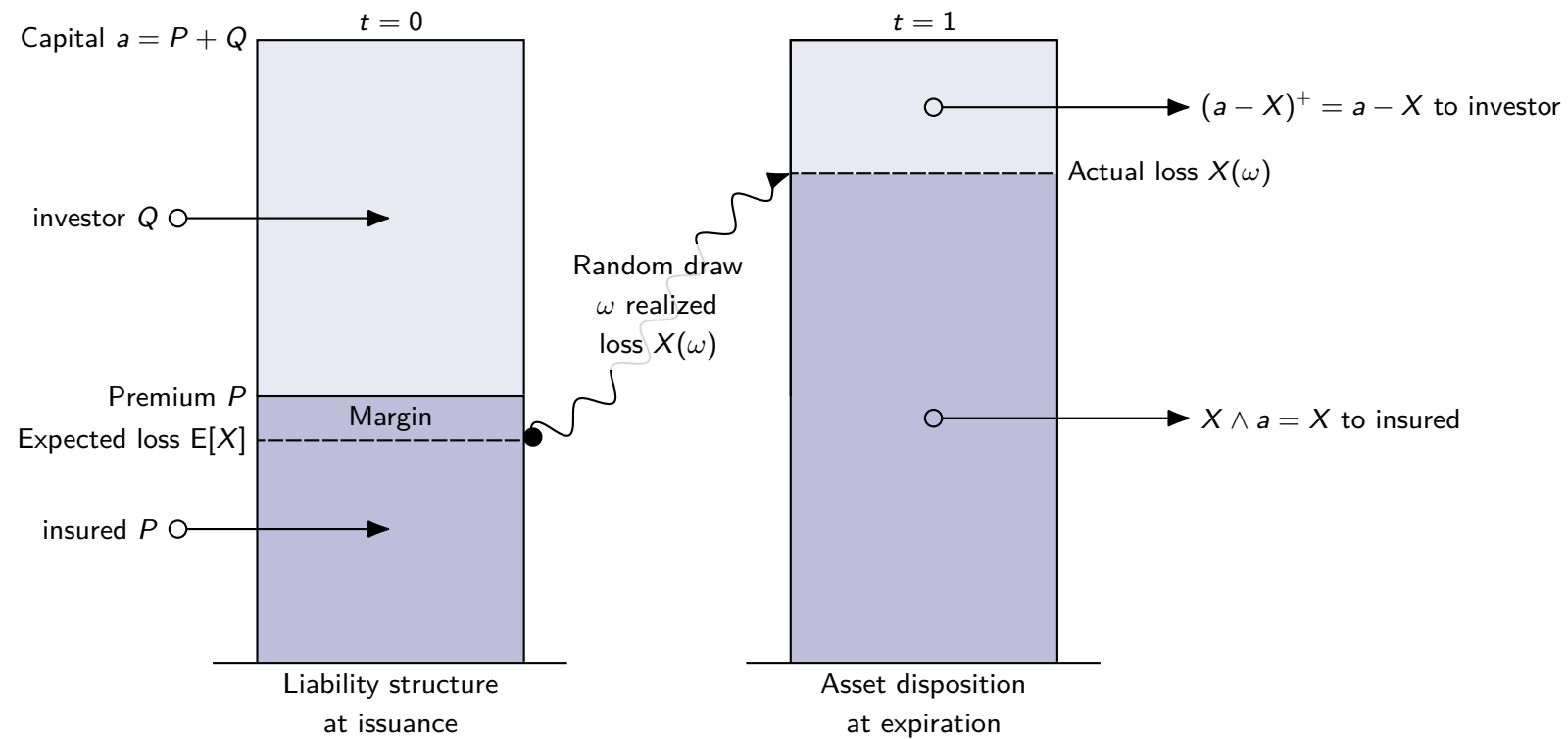
### 3. No taxes

- Tax treatment of different types of capital (debt, reinsurance tax deductible) needs to be considered

### 4. One-period (ticky)

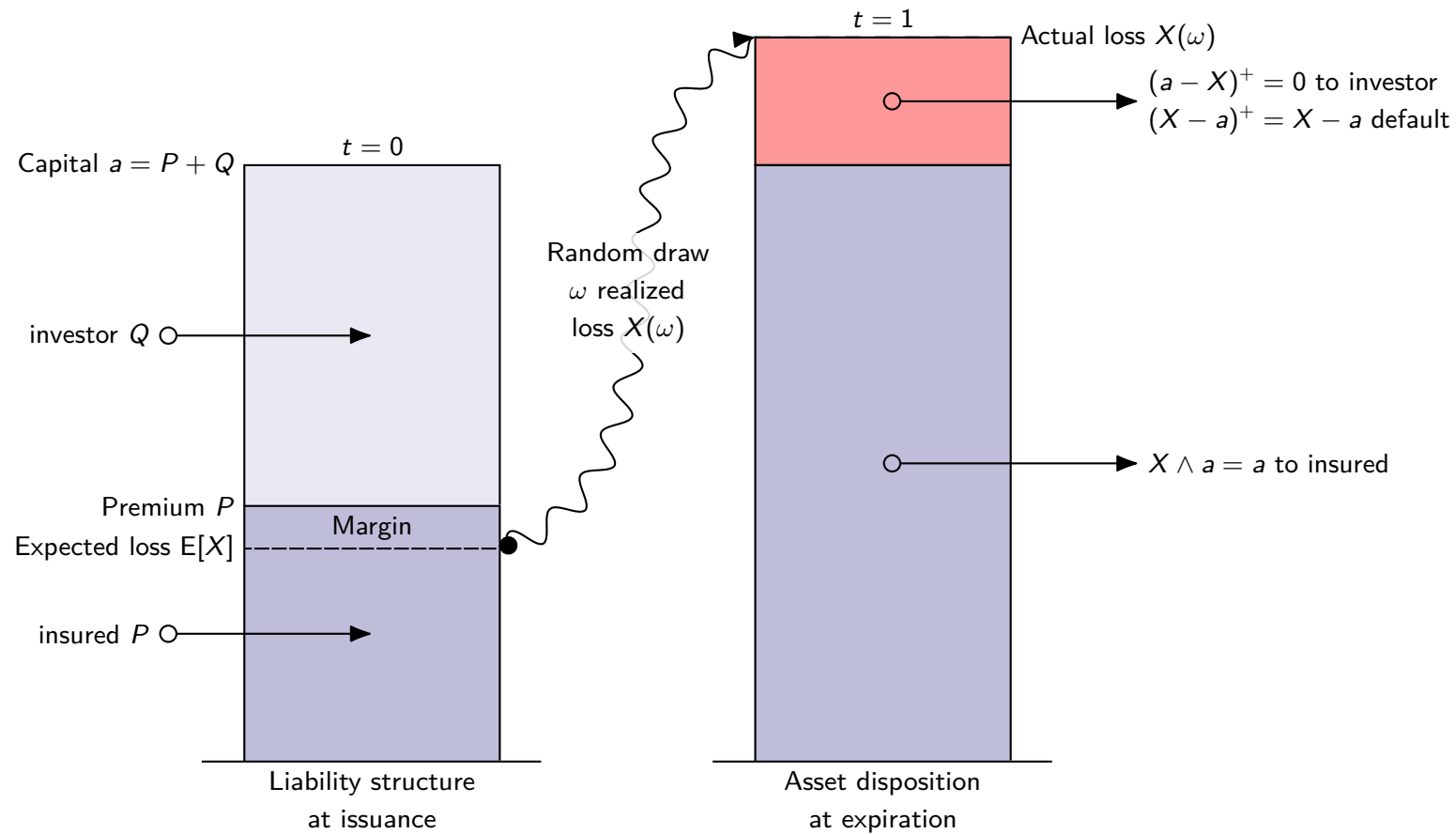
- One-period model implies losses are known with certainty at  $t = 1$  and there are no reserves
- Huge simplification
- Walk before you can run: need to solve the simpler one-period problem first

# Simple Market Model: Initial and Final Transactions in a One-Period Model



- At  $t = 0$  the insured purchases a policy for premium  $P$  and an investor purchases the residual value for paid-in capital  $Q$ , creating starting liabilities (left)
- Asset side of balance sheet is boring: all assets are cash
- At  $t = 1$ , the state of the world  $\omega$  is revealed determining losses, and assets are distributed to the insured and investor, discharging liabilities

# Simple Market Model: Initial and Final Transactions With an Insolvent Outcome



- In an insolvent state,  $X(\omega) > a$  and the insurer defaults
- The insured is paid  $a$  and the investor receives nothing
- Unpaid losses revert to the insured or are covered by a guaranty fund or other socializing mechanism

# Simple Market Model

## Three obvious questions

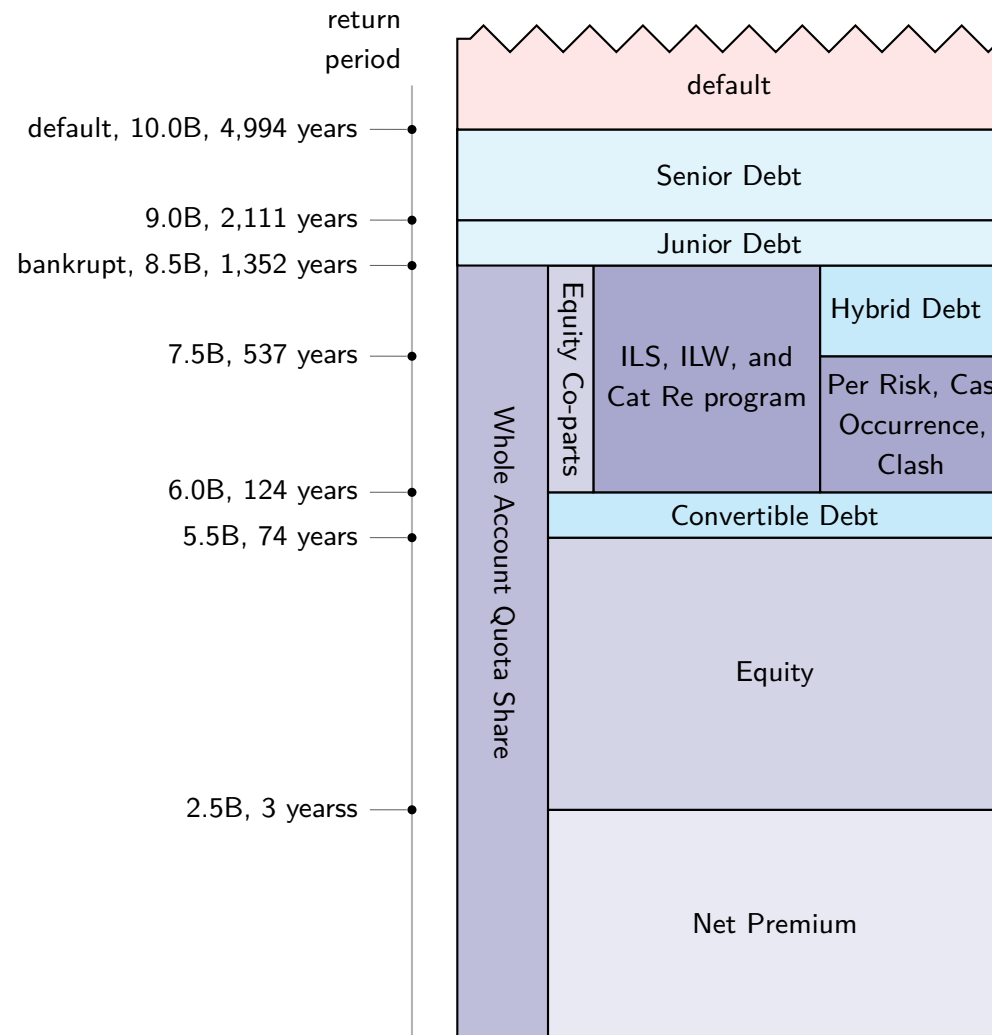
1. What is the appropriate level of assets  $a$ ?
2. How is asset funding split between premium and capital?
3. Are there other forms of capital than an owner purchasing the entire residual value?



# Accounting Definitions

- **Assets** are things you own or are owed
  - An asset is created when a promise is received in exchange for a consideration paid
  - We will assume all considerations are cash
- **Liabilities** are things you owe
  - A liability is created when a promise is made in exchange for cash received
- **Equity** equals assets minus liabilities to all parties except owners
  - Hence: shareholder's equity
- **Capital** equals assets minus all liabilities owed to policyholders, or more senior to policyholders
  - Capital is a legal minimum required to ensure discharge of liabilities, hence minimum capital standards and risk-based capital standards
  - Surplus is used as a synonym for capital in the US
- Debt subordinate to policyholders creates capital but not equity, e.g., surplus notes
- In a one-period model premium is part of capital—see Appendix

# Many Types of Risk Capital



**Figure 1:** Capital structure combining equity, debt, and various forms of reinsurance.

## On- and off-balance sheet capital

- On-balance sheet
  - Equity
  - Preferred equity
  - Debt
- Off-balance sheet capital
  - Reinsurance
  - ILW, ILS
  - Contingent capital
  - Often acts to transform loss
  - Often acts on portion of business
- On-balance sheet equivalence of off-balance assets
  - Cat re has clear limit and equivalence
  - Quota share re does not

# Why Equity is Special

## Non-equity capital has a pre-negotiated cost

- All non-equity forms of risk capital have an explicit, and generally fixed, up-front **cost**
- Debt coupon
- Reinsurance rate, estimated expected recoveries
- Etc.

**Equity has no guaranteed or pre-agreed return**

## Non-equity cost characteristics

- More remote layers, with higher return period of attaching, have lower dollar cost but higher cost per unit of expected loss
- Debt coupon spreads over risk-free by credit rating
- Cat bond and cat reinsurance pricing

# Cost of Capital vs. Cost of Equity

## Weighted Average Cost of Capital (WACC)

Form of Capital	Amount	Cost
Senior Debt	500	0.035
Junior Debt	500	0.055
Reinsurance	1,000	0.085
<b>target ROE</b>	<b>2,500</b>	<b>0.15</b>
<b>Total</b>	<b>4,500</b>	<b>0.112</b>

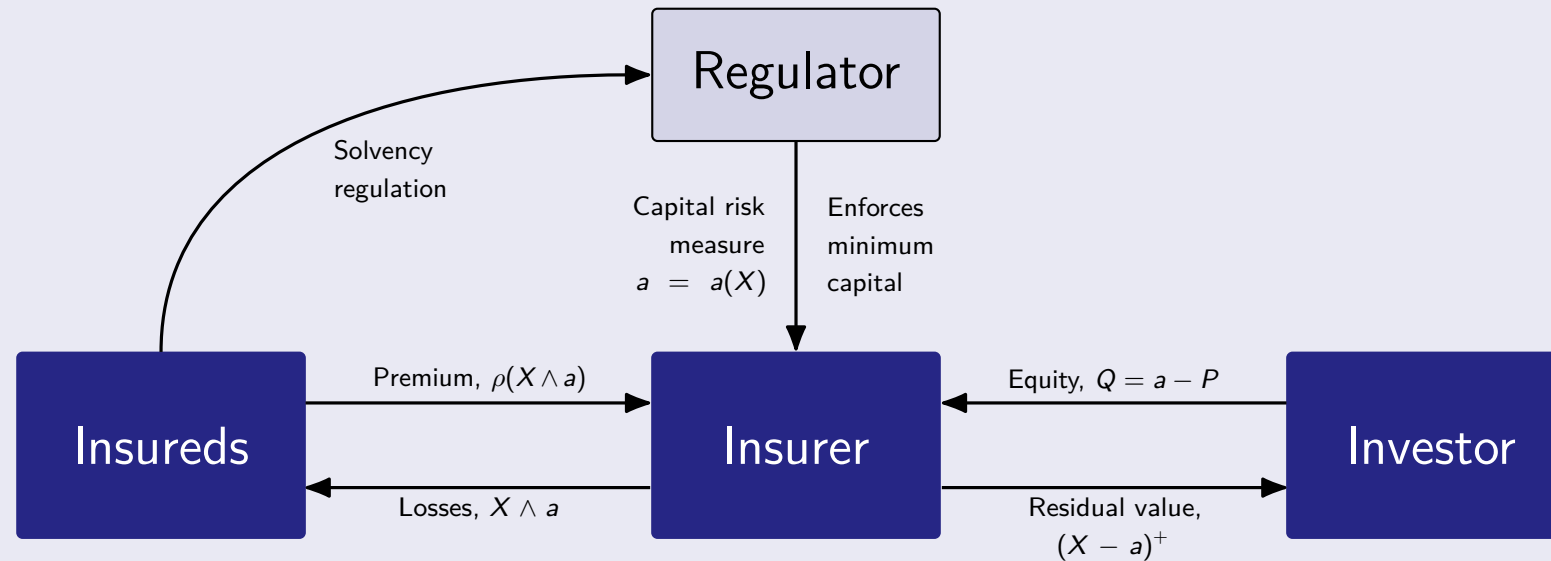
- Costs suitably tax-adjusted: reinsurance and debt are tax deductible
- Reinsurance costs net of expected recoveries

## Target ROE

- Target ROE calibrated as **return needed to support reasonable price-to-book valuation**
- Ambiguous—ties to stock valuation
- Spread over risk-free rate
- Controls cost of retained risk

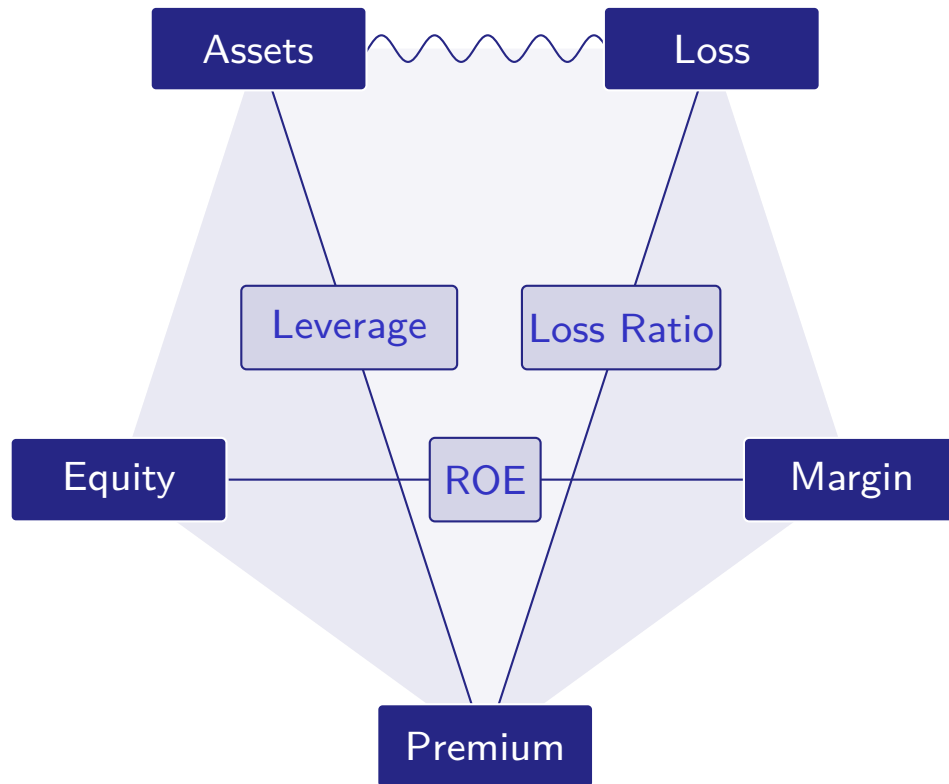
# What Is the Appropriate Level of Assets?

## Regulators or rating agencies determine or heavily influence asset levels



1. Regulator sets required asset standard for risk  $X$
  2. Assets funded through premium from insured with capital paid-in by the investor/owner
  3. Premium  $P$  purchases a promise to pay losses
  4. Capital  $Q$  purchases the residual value of the insurer
  5. The insurer and investor have liability limited
- Two risk measures govern the split of  $a$  into  $P$
1. The regulator evaluates risk  $a(X)$  on an unlimited basis because they are concerned about exposure above insurer assets
  2. The pricing risk measure is evaluated on limited losses,  $\rho(X \wedge a)$

# Insurer Management: Conflicting Objectives



- Five monetary values
- Three ratios underlying
- Three degrees of freedom for insurance pricing

## Four insurance market constraints

1. **Available:** ROE sufficient to attract risk capital
2. **Affordable:** Loss ratio reasonable for buyer
3. **Effective:** Assets cover all reasonable losses
4. **Efficient:** Diversification allows capital leverage

Objectives conflict: more effective, less affordable or available; more affordable less available, etc.

# Two Paradigms of Insurance Pricing

## Financial and actuarial pricing

Product design determines the loss random variable and regulation sets capital standards given losses, leaving only one degree of freedom

1. Actuarial pricing focuses on the **loss ratio** as the free variable
2. Financial pricing focuses on the **ROE**

# Ingredients to Mix an Insurance Pricing Model

## Three ingredients specify an insurance pricing model

1. Total **amount** of capital
2. Total **cost** of capital (ROE)
3. **Allocation** of cost of capital by line

## Common variants

1. Total **amount** of capital
  - a. **VaR** standard very common
  - b. **0.996 VaR** for all methods
2. Total **cost** of capital (ROE)
  - a. **Fixed** over all capital liabilities, e.g., if all capital is equity
  - b. **Varies** over capital liabilities, e.g., with probability of default controlled by a **distortion** function
3. **Allocation** of cost of capital by line
  - a. **Stand-alone** no allocation, not additive
  - b. **Traditional** methods defined by ad hoc allocation rule
  - c. Unique **natural allocation** associated with a distortion

Traditional: single cost of capital, multiple allocations  
Distortion: varying cost of capital, unique allocation



## **Appendix B.I. Accounting and Priority Details**

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# Technical Loss Ratios vs. Combined Ratios

- Net of expenses technical loss ratio  $l$  combined with expense ratio  $e$  produces a combined ratio  $cr = l(1 - e) + e$
- To facilitate comparison with more familiar expense-loaded combined ratios table shows mapping from technical loss ratios LR to combined ratio for different expense ratios

**Table 2:** Impact of expenses on underwriting margin.

LR and ER	0.15	0.2	0.25	0.3	0.35	0.4	0.45
0.25	0.3625	0.4	0.4375	0.475	0.5125	0.55	0.5875
0.35	0.4475	0.48	0.5125	0.545	0.5775	0.61	0.6425
0.45	0.5325	0.56	0.5875	0.615	0.6425	0.67	0.6975
0.55	0.6175	0.64	0.6625	0.685	0.7075	0.73	0.7525
0.65	0.7025	0.72	0.7375	0.755	0.7725	0.79	0.8075
0.75	0.7875	0.8	0.8125	0.825	0.8375	0.85	0.8625
0.85	0.8725	0.88	0.8875	0.895	0.9025	0.91	0.9175
0.95	0.9575	0.96	0.9625	0.965	0.9675	0.97	0.9725

# Accounting Definitions

## Premium as Capital

- In a one-period model premium is part of capital
- All premium is earned before losses are revealed and is available to pay claims
- In reality, interim losses and accounting evaluations lead to the introduction of earned premium, complicating the picture
- Whether unearned premium is available to pay claims is a question of law
  - When unearned premium ranks with losses, it is **not** available to pay claims
  - When it ranks below claims and it is available

States where premium is not available to pay claims: AL, AZ, CA, CO, CT, DE, HI, IA, IL, KS, KY, MT, NC, NH, NJ, NY, OH, OK, OR, PA, RI, TX, UT, VT, WA, WV, WY

States where it is: AK, AR, DC, FL, GA, ID, IN, LA, ME, MI, MN, MO, MS, ND, NE, NM, NV, SC, SD, TN and WI

Unclear: MA, MD and VA; details in appendix

## Premium accounting

- Accounting for premium involves four accounts
  1. Written premium (income statement, revenue)
  2. Change in UPR (income statement, revenue)
  3. Unearned premium reserve (liability)
  4. Premium receivable (asset)
- Income statement accounts flow into equity on the liability side of the balance sheet
- A policy with written premium 1000 is booked as
  - Dr Premium receivable / Cr Written premium, creating income and an asset
  - Dr Change in UPR / Cr UPR, deferring income and setting up a liability
- When premium is collected
  - Dr Cash / Cr Premium receivable, re-arranging assets
- When premium is earned
  - Dr UPR / Cr Change in UPR, unwinding the deferral and creating revenue; available to pay claims

# Default Priorities by State

State	AK	AL	AR	AZ	CA	CO	CT	DC	DE	FL	GA	HI	IA	ID	IL	IN	KS	KY	LA	MA	MD	ME	MI	MN	MO	MS
Expenses	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			1	1	1	1	1
Guaranty Fund				2	2		2		2																	
Claims	3	2	2	3	2	2	3	2	3	2	2	2	2	2	4	2	2	3	2			2	2	2	2	2
Unearned prem	4		3					5		3	5			3		5			5			3	5	3	5	3
Federal Govt	5		4	4	3	3	4	3	4	4	3	3	3	4	6	3	3	4	3			4	3	4	3	4
Employee	2		5	5		4	5	4	5	5	4	4	4	5	3	4	4	5	4			5	1	5	4	5
Reinsurers	4						6	5	6		5		5				5		5			6			5	6
Surplus notes	7		9	9	8	7	8	7	8	9	8	8	8	9	8	8	8	10				10	8	10	8	9
Shareholders	8		10	10	9	8	9	8	9	11	9	9	9	10	9	9	9	11				11	9	11	9	10

State	MT	NC	ND	NE	NH	NJ	NM	NV	NY	OH	OK	OR	PA	RI	SC	SD	TN	TX	UT	VA	VT	WA	WI	WV	WY
Expenses	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Guaranty Fund											2			2					2		2				
Claims	2	2	2	2	2	4	3	2	2	2	3	2	2	3	2	3	2	2	3		3	2	2	2	3
Unearned prem			5	5			4	3							3	4	5						6		
Federal Govt	3	3	3	3	3		5	4	3	3	4	3	3	4	4	5	3	3	5		4	3	3	3	4
Employee	4	4	4	4	4	2	2	6	4	4	5	4	4	5	5	2	4	4	6		5	4	5	4	2
Reinsurers		5	5	5			5		6									5	7			5		5	
Surplus notes	8		8	8	9		9	11	8	8	9		7	8	9	7	8	9	11		9	8	11	8	5
Shareholders	9		9	9	10		10	12	9	10	10		8	9	10	8	9	11	13		10	9	12	9	6

**Table 3:** Priority of each claimant class in each state's priority law. Source: insurance code in each state.

# Insurance Pricing: Three Degrees of Freedom

**Table 4:** There are eight variables germane to insurance pricing, with five relationships between them, leaving three degrees of freedom. Losses are known from product design and regulators strongly influence assets, leaving just one free variable.  $\omega$  represents a random state of the world.

	Variable	Symbol	Units	Value at Issue	Ultimate Value
1.	Loss	$X$	monetary random variable	$E[X]$	$X(\omega) \wedge a$
2.	Margin	$M$	monetary random variable	$M$	
3.	Premium	$P$	monetary amount	$P = E[X] + M$	
4.	Capital	$Q$	monetary random variable	$Q = a - P$	$(a - X(\omega))^+$
5.	Assets	$a$	monetary amount	$a$	
6.	Loss Ratio		ratio	$E[X]/P$	$X(\omega)/P$
7.	ROE		ratio	$M/Q$	
8.	Leverage		ratio	$P/Q$	